Final exam

6-8 Tues. 15 Dec.

Office hours

2-3pm Mathskeller

Tues. 15 Dec.

Review Session.

Monday in Kastle 213

4-5$^{\underline{30}}$

I have enjoyed this class.

C4.4 #8

Find the critical points of $f(x)$ and use the Second Derivative Test (if possible) to determine whether each corresponds to a local minimum or maximum. Let

$$f(x) = xe^{-x^2}$$

Note: The function exp is another name for exponential function with base e. Thus, $\exp(t) = e^t$.

You must enter your critical points in ascending order.

Critical Point 1 = [ ] is what by the Second Derivative Test ? ?

Critical Point 2 = [ ] is what by the Second Derivative Test ? ?

Critical point if $f'(x) = 0$ or if f is not differentiable at x.

$$f'(x) = 1 \cdot e^{-x^2} + x(-2x)e^{-x^2}$$

$$= (1-2x^2)e^{-x^2}$$

$f'(x) = 0$ if $1 - 2x^2 = 0$

or if $x = \pm 1/\sqrt{2}$.

2nd deriv. test.

$f'(x)=0$, $f''(x)>0$ $\smile$ local min.

$f'(x)=0$, $f''(x)<0$ $\frown$ local max.

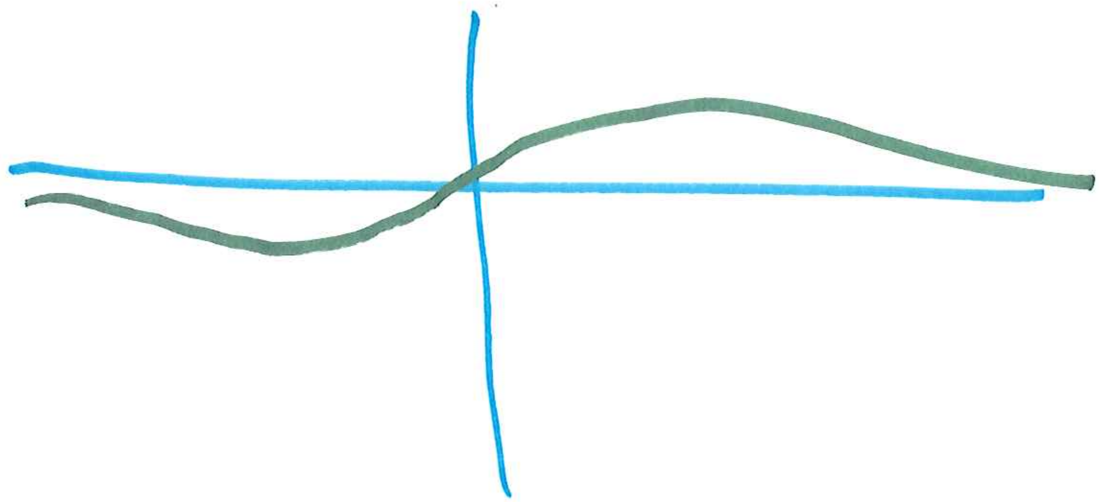
$$f''(x) = \left((1-2x^2)e^{-x^2}\right)'$$

$$= -4x e^{-x^2} + (1-2x^2)(-2x)e^{-x^2}$$

$$= (-6x + 4x^3)e^{-x^2}$$

$f''\left(-\frac{1}{\sqrt{2}}\right) \cancel{<0} > 0$ local min

$f''\left(\frac{1}{\sqrt{2}}\right) \cancel{>0} < 0$ local max.

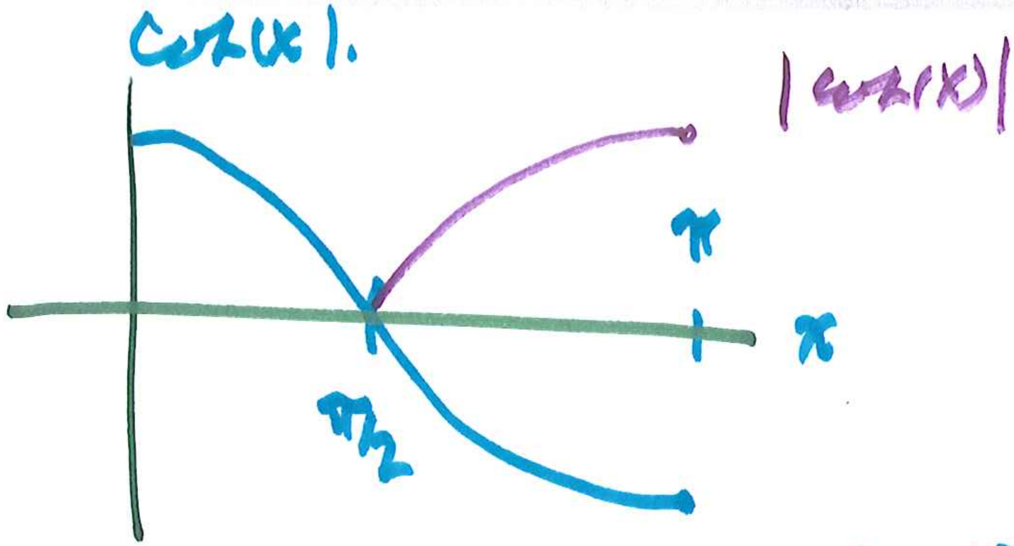


Sketch of graph of xe^{-x^2}.

5.3 and 5.4 #6

Write the integral as a sum of integrals without absolute values and evaluate:

$\int_{\pi/6}^{\pi} |\cos x| \, dx =$



$$|\cos(x)| = \begin{cases} \cos(x) & (0 \le x \le \frac{\pi}{2} \\ -\cos(x) & (\frac{\pi}{2} \le x \le \pi \end{cases}$$

$$\int_{\pi/6}^{\pi} |\cos(x)| \, dx = \int_{\pi/6}^{\pi/2} \cos(x) \, dx + \int_{\pi/2}^{\pi} -\cos(x) \, dx$$

$$= \sin(x)\Big|_{\pi/6}^{\pi/2} - \sin(x)\Big|_{\pi/2}^{\pi}$$

$$= \sin\left(\frac{\pi}{2}\right) - \sin\left(\frac{\pi}{6}\right)$$

$$- \sin(\pi) + \sin\left(\frac{\pi}{2}\right)$$

$$= 1 - \frac{\cancel{\sqrt{3}}\,1}{2} - 0 + 1.$$

$$= 2 - \frac{\cancel{\sqrt{3}}\,1}{2}. = 3/2.$$

5.3-5.4 #15

Let the function F be defined by

$$F(x) = \int_{-7}^{x} (t+2)(t-10)\, e^{-t^2}\, dt.$$

Give the largest interval(s) for which F is decreasing.

Give the largest interval(s) for which F is increasing.

If there is more than one interval, separate the intervals with a comma. Enter NONE if there are no intervals.

$F'(x) = (x+2)(x-10)\, e^{-x^2}$

Need to know if $F'(x) > 0$ or < 0

Need to know if $(x+2)(x-10) > 0$ or < 0

$-2 \qquad 10$

$x+2$	$-$	$+$	$+$
$x-10$	$-$	$-$	$+$
	$+$	$-$	$+$

$F'(x) > 0$ on $(10, \infty)$ and $(-\infty, -2)$. (Inc.)

$F'(x) < 0$ so F is decreasing on $(-2, 10)$.

5.5 #5

A particle moves in a straight line with velocity $12 - 2t$ ft/s. Find the total displacement and total distance traveled over the time interval $[0, 8]$.

Displacement: ____ ft.

Distance: ____ ft.

Displacement is

$$\int_0^8 (12-2t)\,dt$$

$$= 12t - t^2 \Big|_0^8$$

$$= 96 - 64 = 32 \text{ ft}$$

Distance travelled is

$$\int_0^8 |12-2t|\,dt$$

$12-2t>0 \iff 12>2t$

$6>t.$

$12-2t<0 \iff \cancel{6<t}.$

$6<t$

$$\int_0^8 |12-2t|\,dt$$

$$= \int_0^6 12-2t\,dt + \int_6^8 2t-12\,dt$$

$$= 12t-t^2\Big|_0^6 + t^2-12t\Big|_6^8$$

$$= 72-36 + (64-96)$$

$$-(36-72).$$

$$= 36 + -32+36$$

$$= \underline{40}\text{ ft.}$$

$12 - 2t < 0$ if $t > 6$

then

$$|12 - 2t| = -(12 - 2t)$$

$$= \underline{2t - 12}.$$

5.6 #2

Evaluate the indefinite integral. / Anti-derivative

(Use symbolic notation and fractions where needed.)

$\int \frac{x}{\sqrt{x^2+2}}\,dx =$ [ ] $+ C$.

$u = x^2+2, \quad du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$$\int \frac{x}{\sqrt{x^2+2}}\,dx = \int \frac{1}{\sqrt{u}}\,\frac{1}{2}du$$

$$= \frac{1}{2}\int u^{-1/2}\,du = \cancel{\frac{1}{2}}\,\frac{u^{1/2}}{\cancel{1/2}}$$

$$= \sqrt{u} + C'$$

$$= \sqrt{x^2+2} + C.$$

Check.

$$\frac{d}{dx}\sqrt{x^2+2} \stackrel{?}{=} \frac{x}{\sqrt{x^2+2}}.$$

5.8 #4

Find the solution to

$$\frac{dy}{dt} = 7y$$

satisfying

$$y(1) = 2$$

$y =$

$$y(t) = Ae^{7t}$$

$$y(1) = Ae^{7} = 2.$$

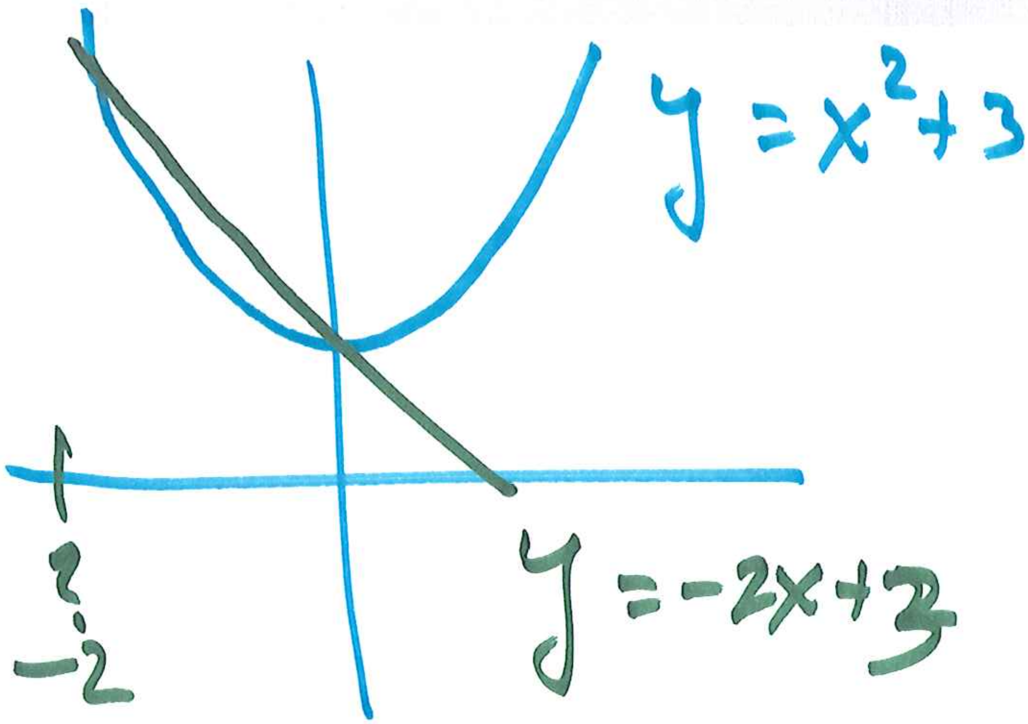
$$A = 2e^{-7}.$$

$$y(t) = Ae^{7t} = 2e^{-7}e^{7t}$$

$$= 2e^{7(t-1)}.$$

D6.1 #1

The region between the graphs of $f(x) = x^2 + 3$ and $g(x) = -2x + 3$ has area [ ] square units.



~~Area~~ Solve $-2x + \cancel{3} = x^2 + \cancel{3}$

$x^2 + 2x = 0$

$x(x+2) = 0$

$x = 0$ or $x = -2$

Area is

$$\int_{-2}^{0} g(x) - f(x)\, dx$$

$$\cancel{= \int_{-2}^{0} |g(x) - f(x)|\, dx}$$

$$= \int_{-2}^{0} (-2x+3) - (x^2+3)\, dx$$

$$= \int_{2}^{0} -2x - x^2\, dx$$

$$= -x^2 - \frac{x^3}{3} \Big|_{-2}^{0}$$

$$= 0 - \left(-4 + +\frac{8}{3}\right)$$

$$= 4 - 8/3 = \frac{12}{3} - \frac{8}{3} = \frac{4}{3}.$$