Calculus I — Russell Brown
Final exam — 13 December 2001

Answer all of the following questions. Additional sheets are available if necessary. No books or notes may be used. You may use a calculator. You may not use a calculator which has symbolic manipulation capabilities. When answering these questions, please be sure to 1) check answers when possible, 2) clearly indicate your answer and the reasoning used to arrive at that answer *(unsupported answers may not receive credit).*
Each question is followed by space to write your answer. Please lay out your solutions neatly in the space below the question. You are not expected to write each solution next to the statement of the question.
The total on this test is 104 points, but all scores be truncated at 100.

Name _____

Section _____

Question	Score	Total
1		16
2		12
3		12
4		8
5		8
6		8
7		16
8		12
9		12
min(Total,100)		100

1. Differentiate the following functions with respect to x. Simplify your answers.

   (a) $x^2 \sin(x)$

   (b) $\sqrt{2x^2 + 1}$

   (c) $\sin(x^2)$

   (d) $\dfrac{\cos x}{2 + \sin(x)}$

2. A boat is pulled into a dock by a rope attached at the bow of the boat and passing through a pulley that is 2 meters higher than the bow of the boat. If the rope is pulled in at a rate of 1 meter/second, how fast is the boat approaching the dock when it is 8 meters from the dock?


1m/s
8m

3. (a) Set up and solve a minimization problem to find the point on the line $y = 2x - 3$ that is closest to the point $(-1, -2)$.

   (b) Find the slope of the line $y = 2x - 3$. Find the slope of the line through $(-1, -2)$ and your answer to part a). Check your answer to part a) by verifying that these lines are perpendicular.

4. (a) Write the limit

$$\lim_{n\to\infty} \sum_{k=1}^{n} \frac{\pi}{n} \sin(\frac{k\pi}{n})$$

as an integral.

(b) Evaluate the limit in part a).

5. State part 1 and part 2 of the fundamental theorem of calculus.

6. Give the interval(s) for which the function F is increasing. The function F is defined by

$$F(x) = \int_0^x \frac{5t - 3}{(t^2 + 10)^2}\, dt.$$

7. Evaluate the integrals:

   (a) $\int_1^2 (6x^2 + x^3)\, dx$

   (b) $\int_0^3 |x - 1|\, dx$ Hint: Draw the graph of the function $|x - 1|$ and use simple facts from geometry to determine the integral.

   (c) $\int_0^{\pi/4} \dfrac{\sin(x)}{\cos^2(x)}\, dx$

   (d) $\int_{-\pi^2}^{\pi^2} \sin^{101}(x^{201})\, dx$

8. Find the area between the curves $y = x^2$ and $y = 1 - x^2$.

9. Compute the volume of the region obtained by rotating the region between the $y = 1$ and $y = x^2$ about the line $y = 3$. Use the method of washers. In your solution, you should a) sketch the region to be rotated, b) state the interval and axis which you will partition, c) sketch a typical strip that will be used to approximate the region, d) give the volume of the washer that is obtained when the strip in part c) is rotated about the line $y = 3$, e) write the total volume as an integral and f) evaluate the integral to find the volume.

   (I believe that this problem is most easily done using washers. You may use shells, if you prefer. For either method, you must carry out the steps described above. Substitute "shell" for "washer" in part d), if necessary.)