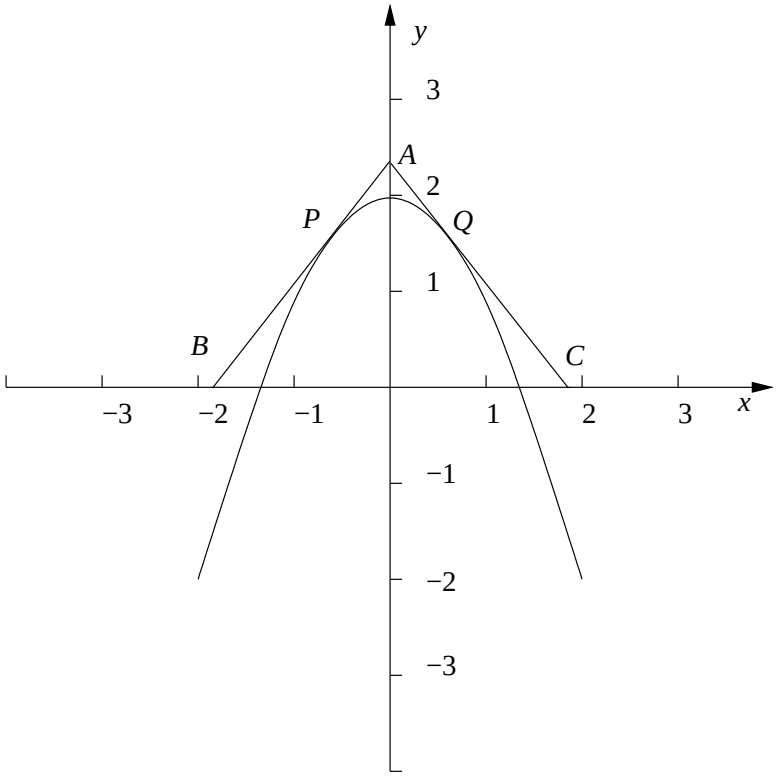
Calculus I *Spring 2003*
MA113:10-12 *Russell Brown*
Homework C

The following is a series of problems related to tangent lines to parabolas. Please write up the solutions carefully, including complete explanations. In your solutions, it is not so important to give every detail of the calculations. Rather, a simple explanation of the main steps. For example if you must solve a quadratic equation, I would like to see you write "Solving the equation (*) by completing the square gives that the roots are:" and then list the roots. I do not need to see every step of algebra along the way. You will be expected to work on this assignment in parallel with your regular classwork. This assignment will be worth 25 homework points of which 10 points will be defined to be extra credit.

1. Consider the parabola $y = 2 - x^2$ and find points P and Q on the parabola so that the tangent lines through P and Q form an equilateral triangle with the x-axis. Hint: What can you say about the slopes of the tangent lines?



2. If we consider the parabola $y = x^2$, and a point (c, d) in the plane, how many tangent lines to the parabola are there that pass through (c, d)? The exercises below answer this question and allow you to give the number of tangent lines in terms of the location of the point.

This problem was stolen from Jim Brennan.

(a) Make three sketches which show the tangent line(s) to $y = x^2$ which pass through through

  i. $(1, -2)$

  ii. $(1, 1)$

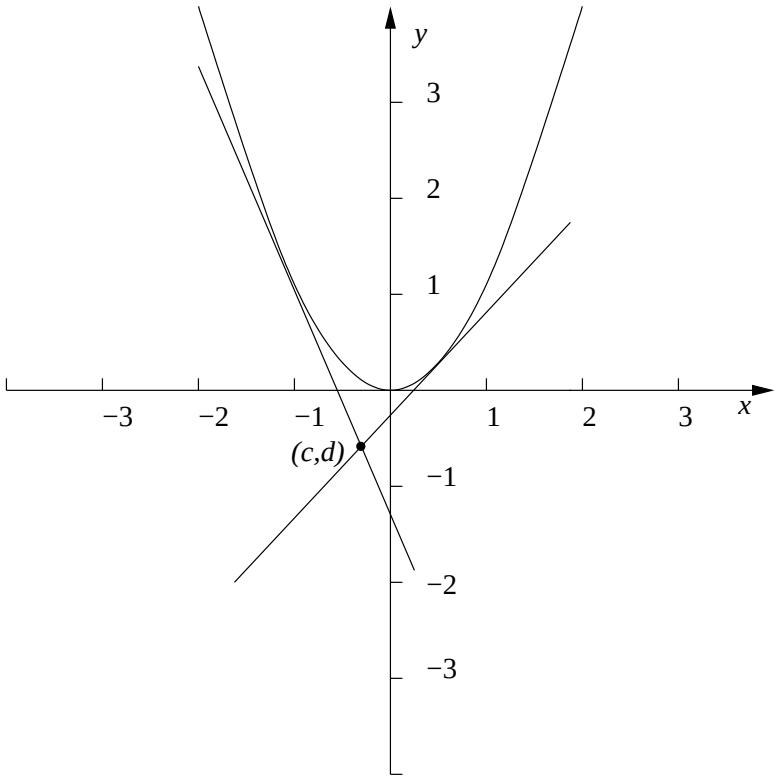
  iii. $(0, 1)$.

(b) Make a conjecture as to how many tangent lines of the parabola pass through a given point (c, d). How does the answer depend on the point (c, d)?

(c) Write the equation of the tangent line to the parabola $y = x^2$ at (x_0, x_0^2).

(d) If we require the tangent line in part c) to pass through the point (c, d), we obtain an equation for a the x-coordinate of the point where the tangent line meets the parabola. Write out this equation.

(e) Tell me how many solutions the equation you found in part d) has. How does the number of solutions depend on (c, d)? The best answers to this question will refere to the discriminant of a quadratic equation. Can you interpret your answer geometrically?



February 10, 2003