

Below are a selection of problems to help prepare you for exam 2. In addition to these problems, you should understand all of the (non-starred) homework problems. Please know the trigonometric identities:

$$\begin{aligned}\sin(x+y) &= \sin(x)\cos(y) + \cos(x)\sin(y) \\ \cos(x+y) &= \cos(x)\cos(y) - \sin(x)\sin(y) \\ 1 &= \cos^2(x) + \sin^2(x)\end{aligned}$$

1. If $\sin(\theta) = 3/5$ and $\pi/2 < \theta < 3\pi/2$, find the remaining trigonometric ratios.

2. Find the limits:

$$\lim_{\theta \rightarrow 0} \frac{1 - \cos \theta}{\theta^2} \quad \lim_{\theta \rightarrow 0} \frac{\tan \theta}{3\theta} \quad \lim_{\theta \rightarrow 2} \frac{\tan \theta}{3\theta}.$$

3. If $f(x)$ is even, what can you say about $\sin(f(x))$?

4. If $f(x)$ is even, what can you say about $f'(x)$?

5. Compute the derivatives of

$$\sin(\tan(\cos(\sec(x^2 + 1)))) \quad \frac{1}{\sin^2 x + \cos^2 x}.$$

6. Compute the derivatives of the following functions:

$$\frac{\tan(2x)}{x} \quad \sin(\cos(x^2)) \quad \cos x \tan x.$$

7. When is the tangent line to

$$f(x) = \sin x + \cos x$$

horizontal?

8. Find all tangent lines to the ellipse $3x^2 + y^2 = 25$ when $y = 4$.

9. Compute the second derivative

$$\frac{d^2}{dx^2} \frac{1}{x^2 + 1}.$$

Simplify.

10. Compute $f^{(101)}(x)$ if $f(x) = \sin(x)$.

11. An airplane is flying due north at 300 km/hour at an altitude of 10 kilometers. Suppose that the airplane is directly above a point 20 kilometers north of an observer. What is the rate of change of the angle of elevation changing? Is it increasing or decreasing?

12. Blake and Piper are standing at the Washington monument. At time $t = 0$, Blake begins running north at 5 kilometers/hour and Piper begins running west at 6 kilometers/hour. How fast is the distance between them increasing after 30 minutes?
13. If an object is moving along the x -axis so that its position at time t seconds is $x(t) = t^3 - 22t^2$ centimeters to the right of the origin, find out when the acceleration is equal to 15.
14. Find a polynomial $P(x)$ so that $P(0) = 1$, $P'(1) = 2$ and $P''(2) = 3$.
15. Find the linear and quadratic approximations to $\cos(x)$ at 0. Use these approximations to estimate $\cos(3^\circ)$, $\cos(0.1)$.
16. Find the linear and quadratic approximations to $\sqrt[3]{x}$ at $x = 1000$. Use these approximations to estimate $\sqrt[3]{983}$ and $\sqrt[3]{1011}$. Check if your answers are reasonable.