1 Lecture 8: The derivative as a function.

1.1 Outline

- Definition of the derivative as a function. definitions of differentiability.
- Differentiability implies continuity.
- Example: Finding a derivative.
- Example: Finding tangent lines.
- Examples: Points where a function is not differentiable.

1.2 The derivative

Definition. Given a function f, we may define a new function f', which we call *the derivative of f* by

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h},$$

provided the limit exists.

An equivalent definition that is sometimes useful is

$$f'(x) = \lim_{y\to x} \frac{f(y) - f(x)}{y - x}.$$

A function is *differentiable* at x, if $f'(x)$ exists. Thus the domain of f' is the set of values x so that f is differentiable at x.

A function is *differentiable* on an interval I if f is differentiable for each x in I.

1.3 Differentiability and continuity.

Theorem 1 *If f is differentiable at x, then f is continuous at x.*

1.4 Examples

Example. Find the derivative of $f(x) = \sqrt{x}$.

Example. Let $f(x) = 1/x$. Find all values x where $f'(x) = 4$. Find all value x where $f'(x) = -4$.

Find all tangent lines to the graph of f which are parallel to the line $y = -4x$.

Example. Let $f(x) = \begin{cases} 1, & x > 0 \\ 0 & x \le 0. \end{cases}$ Show that f is not differentiable at 0.

Example. Let $f(x) = |x|$. Where is f continuous? Where is f differentiable?

Example. Let $f(x) = \sqrt[3]{x}$. Give the domain. Show f is not differentiable at 0.

January 31, 2007