Spring 2015 *Russell Brown*
Calculus I *MA 113:011-015*

1. Make the substitution $u = \cos(x)$ to find

$$\int \tan(x)\,dx = \int \frac{\sin(x)}{\cos(x)}\,dx.$$

If we put $u = \cos(x)$, then $du = -\sin(x)\,dx$ and thus

$$\int \tan(x)\,dx = -\int \frac{1}{u}\,du = -\ln(|u|) + C = -\ln(|\cos(x)|) + C.$$

2. Find the antiderivative

$$\int \frac{x}{1+x^2}\,dx.$$

What substitution should we use?

Let $u = 1 + x^2$ and then $du = 2x\,dx$ or $x\,dx = \frac{1}{2}\,dx$.

Substituting gives

$$\int \frac{x}{1+x^2}\,dx = \frac{1}{2}\int \frac{1}{u}\,du = \frac{1}{2}\ln(|u|) + C = \frac{1}{2}\ln(1+x^2) + C.$$

3. Find the definite integral,

$$\int_0^1 \frac{1}{(2+3u)^2}\,du.$$

Use the substitution $v = 2 + 3u$.

If $v = 2 + 3u$, then $dv = 3\,du$. If $u = 0$, then $v = 2$ and if $u = 1$, then $v = 5$. Substituting gives

$$\int_0^1 \frac{1}{(2+3u)^2}\,du = \frac{1}{3}\int_2^5 \frac{1}{v^2}\,dv = -\frac{1}{v}\Big|_{v=2}^5 = \frac{1}{3}(-\frac{1}{5} + \frac{1}{2}) = \frac{1}{10}.$$