Lecture 02: Rational Functions

Russell Brown

Department of Mathematics
University of Kentucky

Question 1.

Find the integral

$$\int_0^1 \frac{1}{1+x^2}\,dx$$

(Enter the letter corresponding to your answer as text.

J 0

K $\pi/6$

L $\pi/4$

M $\pi/3$

N $\pi/2$

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We have

$$\begin{aligned}\int_0^1 \frac{1}{1+x^2}\,dx &= \tan^{-1}(x)|_{x=0}^1 \\ &= \tan^{-1}(1) - \tan^{-1}(0) = \pi/4.\end{aligned}$$

Question 2.

Find the anti-derivative

$$\int \frac{x^2+2}{x^2+1}\,dx.$$

U $x + 2\tan^{-1}(x) + C$

V $2x + 2\tan^{-1}(x) + C$

W $x + \tan^{-1}(x) + C$

X $2x + \tan^{-1}(x) + C$

Y $42 + C$

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U.

We can write

$$\begin{aligned}\int \frac{x^2+3}{x^2+1}\,dx &= \int 1 + \frac{2}{x^2+1}\,dx \\ &= x + 2\arctan(x) + C\end{aligned}$$

Question 3.

Enter the letters (O,P,Q,R,S) corresponding to all terms which appear in the partial fraction decomposition of

$$\frac{x^2+2}{x^5+2x^3+x}.$$

O A/x

P $(Bx+C)/(x^2+1)$

Q $Hx^2/(x^4+2x^2+1)$

R $(Dx+E)/(x^2+1)^2$

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O, P, and R appear in the partial fraction decomposition.