

Case 2

Example.

$$\int_0^{\pi} \sin^3 x \, dx.$$

Solution

$$\int \sin^3 x \, dx = \int \sin^2(x) \sin(x) \, dx$$

$$= \int (1 - \cos^2(x)) \sin(x) \, dx$$

$$\text{Set } du = \sin(x) \, dx$$

$$u = -\cos(x), \quad u^2 = \cos^2(x).$$

$$= \int (1 - u^2) \, du = u - \frac{u^3}{3} + C.$$

$$= -\cos(x) + \frac{\cos^3(x)}{3} + C$$

$$\text{Then, } \int_0^{\pi} \sin^3(x) \, dx = -\cos(x) + \frac{\cos^3(x)}{3} \Big|_0^{\pi}$$

$$= -\cos(\pi) + \cos(0) + \frac{\cos^3(\pi)}{3} - \cos(0)$$

$$= \frac{4}{3}.$$