Lecture 04: Integrating powers of sine and cosine

Russell Brown

Department of Mathematics
University of Kentucky

Question 1.

Which of the following are true for all values of x? While there are several correct answers, it is early in the morning and you only need to select one answer.

G $\cos^2(x) + \sin^2(x) = 1$

H $\cos(2x) = 2\cos^2(x) - 1$

I $\cos^2(x) = \dfrac{1 + \cos(2x)}{2}$

J $\sin^2(x) = \dfrac{1 - \cos(2x)}{2}$

K $\cos(2x) = 1 - 2\sin^2(x)$

L $\sin(4x) = 2\sin(2x)\cos(2x)$

M $\cos(6x) = \cos^2(3x) - \sin^2(3x)$

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G,H,I,J,K,L,M
All statements are identities.

Question 2.

What substitution should you make to evaluate

$$\int \tan(x)\,dx = \int \frac{\sin(x)}{\cos(x)}\,dx?$$

O $u = \sin(x)$

P $u = \cos(x)$

Q $u = \tan(x)$

R $x = \tan(u)$

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P is the best answer. If $u = \cos(x)$, then $du = -\sin(x)dx$. Thus substituting gives

$$\int \frac{\sin(x)}{\cos(x)}\,dx = -\int \frac{1}{u}\,du.$$

Question 3.

Find an anti-derivative.

$$\int \sin(x)\cos(x)\,dx.$$

Several answers are correct, but you only need to choose one.

O $\frac{1}{2}\sin^2(x) + C$

P $-\frac{1}{2}\cos^2(x) + C$

Q $-\frac{1}{4}\cos(2x) + C$

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O, P, and Q are all correct.
To obtain O, use the substitution $u = \sin(x)$. The answer P arises if you substitute $u = cos(x)$. If we use the double-angle formula $\sin(x)\cos(x) = \sin(2x)/2$, we obtain Q.