Lecture 05: A few trig substitutions

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Question 1.

Which of the following are true for all values of x? While there are several correct answers, it is early in the morning and you only need to select one answer.

N
$$\cos^{2}(x) + \sin^{2}(x) = 1$$

O $\tan^{2}(x) + 1 = \sec^{2}(x)$
P $\cot^{2}(x) + 1 = \csc^{2}(x)$
Q $\sin(x) = \sqrt{1 - \cos^{2}(x)}$
R $\cos(x) = \sqrt{1 - \sin^{2}(x)}$



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N,O,P

For some values of x, we have $sin(x) = -\sqrt{1 - cos^2(x)}$, thus Q may fail.



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Question 2.

If $x = \cos(u)$ and $0 \le u \le \pi$, which expression gives $\tan(u)$? F $x/\sqrt{1-x^2}$ G $1/1 + x^2$ H $x/\sqrt{1+x^2}$ I $\sqrt{1-x^2}/x$ J 42



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For this range of u, we have $\sin(u) = +\sqrt{1 - \cos^2(u)} = \sqrt{1 - x^2}$. Thus $\tan(u) = \sin(u)/\cos(u) = \sqrt{1 - x^2}/x$.



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Question 3.

What substitution should you make to evaluate

$$\int \sqrt{9-4t^2} \, dt?$$

T $t = \frac{3}{2}\cos(u)$ U $t = \frac{2}{3}\cos(u)$ V $t = \frac{3}{2}\sin(u)$ W $t = \frac{9}{4}\sin(u)$ X $t = \frac{4}{9}\sin(u)$ Y $t = \frac{4}{9}\cos(u)$ Z $u = \frac{4}{9}\sin(t)$



A b

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$$U t = \frac{2}{3}\cos(u)$$

V
$$t = \frac{3}{2}\sin(u)$$

W
$$t = \frac{9}{4} \sin(u)$$

$$X \ t = \frac{4}{9}\sin(u)$$

$$Y \ t = \frac{4}{9}\cos(u)$$

$$Z u = \frac{4}{9}\sin(t)$$

T or V. If we put $t = \frac{3}{2}\sin(u)$, then we obtain $\sqrt{9-t^2} = \sqrt{9-4 \cdot \frac{9}{4}\sin^2(t)} = 3\sqrt{1-\sin^2(u)}$. Thus V is the expected answer. The substitution T would work as well.