Lecture 06: Numerical integration

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Question 1.

Find the exact value of the integral

$$\int_1^2 \frac{1}{x}\,dx.$$

Type the name corresponding to the correct answer.

John	$3/4$
Paul	$1/2$
George	$\ln(2)$
Ringo	0.693

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From the Fundamental Theorem of Calculus, we have $\int_1^2 \frac{1}{x}, dx = \ln(|x|)|_{x=1}^2 = \ln(2) - \ln(1) = \ln(2)$. The decimal 0.693 is an approximation to the right answer.

Question 2.

Find the right sum R_4 for the integral

$$\int_1^2 x\,dx.$$

Enter the exact answer as a fraction or decimal.

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We have four equal intervals of width $1/4$. The right sum is

$$R_4 = \frac{1}{4}\left(\frac{5}{4} + \frac{6}{4} + \frac{7}{4} + 8/4\right) = 26/16 = 13/8 = 1.625.$$

The original version of this question was not worded correctly, so full credit was given for all numerical responses.

Question 3.

Use the error estimate for the trapezoid rule to find the smallest value of N for which we know that

$$|T_N - \int_1^4 \frac{1}{x}\,dx| \leq 0.01.$$

Enter N as your answer.

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$N = 22$. We continue to have $|f''(x)| \leq 2$ for $1 \leq x \leq 4$. From the error estimate we have

$$|E_T| \leq \frac{2 \cdot 3^3}{12N^2} = \frac{9}{2N^2}$$

Solving the inequality, $\frac{9}{2N^2} \leq \frac{1}{100}$ we obtain $N^2 \geq 450$. The smallest positive integer that satisfies this inequality is $N = 22$ since $\sqrt{450} \approx 21.2$.