

Examples, Review for final

1. Fred Foy has \$100, 000 to invest in stocks, bonds and a money market account. The stocks have an expected return rate of 6% per year, the bonds pay 8% per year and the money market earns 2% per year. Fred insists that the money in the money market funds should equal the sum of 30% of the amount invested in stocks and 10% of the amount invested in bonds. Fred wants to allocate the \$100,000 among the three investments in order to provide an expected income of \$8,000 per year. Use x , y , z to denote the amounts invested in stocks, bonds and money market respectively. Answer the following questions.
 - (a) Write down a linear equation in x , y , z relating to the total investment.
 - (b) Write down a linear equation relating to the expected annual income.
 - (c) Write down additional linear equation(s) as needed to express the remaining conditions on the investments.
 - (d) Construct an appropriate augmented matrix to solve all the linear equations constructed above. It is not necessary to operate on this matrix any further at this point.

2. Perth mining company operates two mines for the purpose of extracting gold and silver. The Saddle mine costs \$14,000 per day to operate and it yields 50 ounces of gold and 3,000 ounces of silver each day. The Horseshoe mine costs \$16,000 per day to operate and yields 75 ounces of gold and 1,000 ounces of silver each day. The company management has a target of at least 650 ounces of gold and at least 18,000 ounces of silver.

Set up a linear programming problem whose solution will determine how many days each mine should be operated to reach the target while minimizing the cost of operation.

You must define all the variables and list all the constraints. It is not necessary to solve the problem.

3. Consider these inequalities and answer the questions below.

$$x+y \leq 10, \quad 2x+3y \leq 20, \quad x \geq 0, \quad y \geq 0.$$

- (a) Graph the feasible set of the above inequalities on the given graph paper.
- (b) List all the corner points of the feasible set. Be sure to mark them on your graph as well.
- (c) Determine the maximum value of $3x + 2y$ on the above feasible set.

4. In a high school there are 400 seniors of which 250 are female. 60% of the females and 40% of the males have their drivers licences. A student is chosen at random from the senior class. Answer the following:
- (a) What is the probability that the chosen student is a female with a drivers licence?
 - (b) What is the probability that the chosen student is a male with a drivers licence?
 - (c) What is the probability that the chosen student has a drivers licence?
 - (d) Given that the chosen student does not have a drivers licence, what is the probability that the student is a male?

5. Set this problem up, by stating the chosen variables, the function to be maximized and all the inequalities. Do not solve the problem. WidgetSS makes and sells pulleys and sprockets. Each pulley sells for \$18, requires 1.5 hours of finishing and 3.5 hours of machining. Each sprocket sells for \$20, requires 2.5 hours of finishing and 1.5 hours of machining. The company has 100 hours of finishing time and 90 hours of machining time available.

Set up a LPP whose solution will determine how many pulleys and how many sprockets should be made to maximize the company profit.

- (a) Define and explain all the variables you use.
- (b) Now describe the LPP explicitly. We have the objective function $P = 18x + 20y$ to be maximized. The conditions are:
- (c) The initial Simplex tableau is:

6. Consider the following system of linear equations.

$$\begin{aligned}x + 2y + z &= 8 \\ 3x + 7y - 3z &= 8\end{aligned}$$

- (a) Write down the augmented matrix for this system of equations.
- (b) Reduce the augmented matrix to REF (the row echelon form). It is essential to show the steps of row reductions and explicitly write the row operations used.
- (c) Using above calculations, determine all the solutions to the system of equations in x , y , z given above.

7. Here is a final tableau associated with a maximal LPP.

x	y	z	s	t	u	P	$constants$
-2	0	1	0	-4	-13	0	6
-1	1	0	0	0	1	0	4
1	0	0	1	1	4	0	2
2	0	0	0	1	9	1	7

- (a) Using your knowledge of the Simplex algorithm, determine the solution to the maximal LPP.
- (b) If we were to read the solution of the dual minimization problem from the same tableau, then its variables would correspond to s , t , u with values 0 , 1 , 9 . Thus the minimization problem solution is: