## 1 Induction

## 1.1 Sums of powers

1. Find the sums:

$$\sum_{k=1}^{5} (2k-1), \qquad \sum_{k=1}^{6} (2k-1), \qquad \sum_{k=1}^{7} (2k-1).$$

2. Guess a value for the sum

$$\sum_{k=1}^{n} (2k-1).$$

3. Find a formula for the sum  $S = 1 + 2 + 3 + \cdots + n$  by adding the expressions

$$S = 1 + 2 + \dots + n.$$
  
 $S = n + n - 1 + \dots + 1.$ 

- 4. Find a simple expression for  $\sum_{k=1}^{n} 1$ .
- 5. Find the value of the sum

$$\sum_{k=1}^{n} (2k-1).$$

6. Use the following picture to find the value of the sum

$$\sum_{k=1}^{n} (2k-1).$$

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$$\cdot \cdot \cdot *$$

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- 7. Use the method of problem 3 to find the value of the sum  $\sum_{k=1}^{n} (2k-1)$ .
- 8. Consider the sum

$$\sum_{k=1}^{n} ((k+1)^2 - k^2).$$

Find a simple expression for the value of this sum.

9. Using problem 8, give another way to evaluate the sum

$$\sum_{k=1}^{n} (2k-1).$$

10. Consider the sum

$$\sum_{k=1}^{n} ((k+1)^3 - k^3).$$

Find a simple expression for the value of this sum.

11. Using problem 10, find a simple expression for the value of the sum

$$\sum_{k=1}^{n} k^2.$$

- 12. Can you use the method of problem 3 for this problem?
- 13. If there are three people in a room and each person shakes every other person's hand, how many handshakes occur?
- 14. If there are four people in a room and each person shakes every other person's hand exactly one time, how many handshakes occur?
- 15. If there are 310 people in a room and each person shakes every other person's hand exactly one time, how many handshakes occur?
- 16. How many diagonals are there in a convex n-gon?
- 17. How many squares are there on a chess board? This includes not just  $1 \times 1$  squares, but also  $2 \times 2$  squares and so on.