## 2 Counting.

In most counting problems, we try to list all of the ways that we can carry out a task. Often it is simpler to break a task up into a number of simple tasks.

## 2.1 Counting words

- 1. How many ways can we arrange the letters in MAT? List all of the arrangements.
- 2. How many ways can we arrange the letters in MATH? List all the arrangements. How can we use the answer to the previous problem to help us answer this question?
- 3. How many ways can we arrange the letters in MATHS? Describe a systematic way to list all the arrangements.
- 4. If we can arrange n letters in  $P_n$  ways, how many ways can we arrange n + 1 letters? Find a formula for  $P_n$ .
- 5. How many two letter words can we form using the 26 letters of the roman alphabet?
- 6. What is the probability that a random two letter word will be acceptable in Scrabble? The number of acceptable scrabble words of various lengths are given in the following table<sup>1</sup>.

Length in letters	Number of words
2	101
3	1015
4	4030
5	8938
6	15778
7	24029

Table 1: Number of words that are acceptable in American English Scrabble.

http://www.research.att.com/~njas/sequences/A124015

7. How many three, four and five letter words can we form using the 26 letters in the roman alphabet? What is the probability that a random string of three letters is acceptable in Scrabble? Consider the same question for four and five letter words.

<sup>&</sup>lt;sup>1</sup>http://www.research.att.com/~njas/sequences/A124015

- 8. Can you estimate the number of 8 letter words that are acceptable in Scrabble?
- 9. CATS are the only four letters in the alphabet used by the Big Blue Nation and all words are four letters long or less. How many words are there?
- 10. How many words are there in the Big Blue Nation if no word can contain repeated letters?
- 11. There are six letters in the alphabet for Froggish. A word contains exactly six letters of which at least two are the same. How many words are there in Froggish?
- 12. We toss a die 4 times. How many outcomes have at least one 6?
- 13. How many ways can we arrange the letters in BOOK? How many ways can we arrange the letters in KOOK?
- 14. How many ways can we arrange the letters in  $A_1LA_2BA_3MA_4$ ? How many ways can we arrange the letters in ALABAMA?
- 15. How many ways can we arrange the letters in MISSISSIPPI?
- 16. We have 27 boys and 27 girls. How many different ways can we arrange them in pairs for a dance?
- 17. How many ways can we seat 11 people at a circular table?
- 18. How many 3 letter words can we form using the letters in BOOK?
- 19. How many 5 letter words can we form using the letters in RUSSELL?