2.2 Connection between counting and the binomial expansion

Is it an accident that the binomial coefficients $\binom{n}{k}$ arise when expanding $(a+b)^n$ and in some counting problems?

1. We flip a coin four times and make a word using H and T to represent the outcomes. How many different words can we form?

2. If we flip a coin 5 times and record the results to produce a five letter word in the letters H and T, how many words have 3 H's and 2 T's? How many words have 2 H's and 3 T's?

3. If we flip a coin n times and record the flips, how many different outcomes can we have? How many of these outcomes have k H's and $n-k$ T's?

4. We have 310 distinct shiny beads. How many pairs of beads are there?

5. If we have a collection of n shiny beads, how many different subsets are there? How many of these subsets contain k shiny beads?

6. How many 10 digit numbers are there?

7. How many 10 digit numbers have at least one digit appearing twice?

8. Expand $(h+t)^2$. Assume the standard properties of addition and multiplication, except we do not assume that multiplication is commutative.

9. Expand $(h+t)^3$. Assume the standard properties of addition and multiplication, but do not assume that multiplication is commutative.

10. Why do the same numbers $\binom{n}{k}$ arise in counting outcomes of coin flips and in expanding the expression $(x+y)^n$?

11. If we flip a three sided coin 15 times, how many possible outcomes are there? If the sides are A, B, and C, how many outcomes have 5 A's 5 B's and 5 C's?

12. If we expand $(x+y+z)^{15}$, what is coefficient of $x^5y^5z^5$?

13. Expand the expression $(x+y+z)^3$.