MA/CS 321:001 Instructor: Russell Brown
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Announcements: In section 4.3, we will cover the subsections titled "First derivative formulas via Taylor series," "Richardson extrapolation" through the paragraph where the words **Richardson extrapolation** appear in bold and "Second derivative formula via Taylor series."

Homework 6, Due Friday, 22 October 2004.

1. (5 points) page 177, #2.

2. (5 points) Suppose $f(x) = \cos(x)$ and p_2 is a quadratic interpolating polynomial for f on $[-1, 2]$ with nodes -1, 0, and 2. Find a number A so that $|f(x) - p_2(x)| \le A$ for x in $[-1, 2]$. Use the first theorem on error in polynomial interpolation. If you wish, you may check your answer in matlab.

3. (5 points) How many equally spaced nodes would we need to find an interpolating polynomial which differs from $\cos(2x)$ by at most 10^{-5} on $[0, \pi/2]$?

4. (15 points) In this problem, we consider a different approach to estimating the error in linear interpolation. Suppose that f is a function and that the second derivative, f'' exists for each x in an interval $[x_0, x_1]$. We let $p_1(x)$ be the interpolating polynomial for f on this interval. Thus $p_1(x_0) = f(x_0)$, $p_1(x_1) = f(x_1)$ and p_1 is linear.

   Also, recall that if f is function which satisfies $f''(x) \ge 0$ ($f''(x) \le 0$) for all x in $[a, b]$, then f is concave up (down).

   (a) Sketch the graph of a function g that is concave down on an interval $[a, b]$ and has $g(a) = g(b) = 0$. This illustrates, but does not prove, that:
   If g is concave down and $g(a) = g(b) = 0$, then $g(x) \ge 0$ for $a \le x \le b$.

   (b) If g is concave up and $g(a) = g(b) = 0$, what can we conclude about $g(x)$ for x in (a, b)?

   (c) Consider $g(x) = f(x) - p_1(x) + M(x_1 - x)(x - x_0)$ where M is a constant. Show that if $2M \le f''(x)$ for x in $[x_0, x_1]$, then g is concave up on the interval $[x_0, x_1]$.

   (d) Find a condition on the value of N so that
   $h(x) = f(x) - p_1(x) + N(x_1 - x)(x - x_0)$ is concave down on $[x_0, x_1]$.

   (e) Suppose that for some number A, we have $|f''(x)| \le A$ for x in $[x_0, x_1]$.
   Establish that

$$-\frac{A}{2}(x_1 - x)(x - x_0) \le f(x) - p_1(x) \le \frac{A}{2}(x_1 - x)(x - x_0), \qquad x \in [x_0, x_1].$$

(f) Find the maximum value of $(x_1 - x)(x - x_0)$ for x in $[x_0, x_1]$ and find a number B (involving A, x_0 and x_1) so that $|f(x) - p_1(x)| \leq B$ for x in $[x_0, x_1]$.

5. (5 points) page 191 #3.

6. (5 points) page 192 #7.

7. (5 points) page 193 #18.

Corrected: October 15, 2004