MA/CS 321:001
MWF 11:00–11:50
FB 213
Fall 2004

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Announcements.
Homework 7 will be due on Monday, 1 November 2004. The exam will be delayed until Friday, 5 November 2004. Please let me know immediately if this causes a conflict.
Before the next exam, we will cover the following subsections in the text:
§5.1 Definite and indefinite integrals, Lower and upper sums, Riemann integrable functions,
§5.2 Description, Uniform spacing, Error analysis, Applying the error formula.
§6.1 Simpson's rule, Composite Simpson's rule.
Additional information on errors in Simpson's rule will be presented in lecture. Take good notes!

Homework 7.

1. (5 points) Write a matlab function `trap` that uses the trapezoid rule with n subdivisions to approximate the integral of f. The first line of your m-file should be `function I = trap(f,a,b,n)`. The parameter `f` will be function handle. Thus the function `trap(@sin, 0, pi/2, 10)` should return an approximation to $\int_0^{\pi/2} \sin x \, dx$.

2. (15 points) Test the function you wrote above by computing the following integrals for 2^N subintervals $N = 1, 2, \ldots, 10$. Compute the error,

$$E(2^N) = \left| \int_a^b f(x)\, dx - \mathtt{trap(f,a,b,}2^N\mathtt{)} \right|.$$

   Determine if the error is proportional to 2^{-2N} as predicted by the theorem on errors for the trapezoid rule. To do this, plot the values of $(\log_2 2^N, \log_2 E(2^N)) = (N, \log_2(E(2^N))$. If the $E(2^N) = A2^{-2N}$, then the values of $(N, \log_2(E(2^N))$ should lie on a line of slope -2. If the plot appears linear, estimate the slope. If the plot does not appear linear, explain why our theorem on errors does not apply.

   (a) $\int_1^3 \sin(x)\, dx$

   (b) $\int_0^2 \sqrt{x}\, dx$

   (c) $\int_0^2 \tan(x)\, dx$.

   Warning/hint: I expect that in one part you will obtain a linear plot of slope -2, in one part the theorem on errors does not apply, and one part of the exercise does not make sense.

For this problem, you should hand in your script which you use to carry out the integrations, store the error and produce the plots. You should hand in two plots, discuss the error, and tell me which one of the parts is silly and why.

3. (10 points) The goal of this problem is to approximate $I = \int_0^\infty e^{-x^2}\,dx$ to three decimal places. The value of this integral is important in the study of probability. The exact value, that is the answer to this question, can be be found in your Calculus III text. We cannot treat this integral directly with the trapezoid rule because we are integrating over a very long interval.

   (a) Convince yourself that $e^{-x^2} \leq e^{-x}$ if $x \geq 1$. Use this inequality and the comparison theorem to find a value of N so that

   $$\int_N^\infty e^{-x^2}\,dx \leq \frac{1}{2}10^{-3}.$$

   Hint: You may want to look up the comparison theorem in your textbook from Calculus II.

   (b) Use the error estimate for the trapezoid rule to find a value of n so that the trapezoid rule with n divisions approximates the integral

   $$\int_0^N e^{-x^2}\,dx$$

   with an error of at most $\frac{1}{2}10^{-3}$. Here, N is the number we found in part a).

   (c) Use the `trap` function you wrote in an earlier exercise to find a value which approximates the integral I with an absolute error of at most 10^{-3}. You will probably want to write a separate m-file `ff` so that `ff(x)` returns the value of $\exp(-x \cdot x)$.

4. (5 points) p. 204 #2.

5. (5 points) Use Simpson's rule to approximate

   $$\int_0^1 \frac{dx}{1+x^2}$$

   using four equal subintervals. Use the exact value of this integral to determine the error.

6. (5 points) How many subdivisions do we need in order to approximate

   $$\int_0^2 e^{2x}\,dx$$

   with Simpson's rule and be sure that the absolute error is at most 10^{-8}? You do not need to actually do the computation.

7. (5 points) Suppose that $p'(0) = p(0) = p(1) = 0$ and that p has derivatives of all orders. What is the largest value of k for which we can be sure that $p^{(k)}(\xi) = 0$ for some ξ in $(0, 1)$.

8. (5 points, extra credit) Find a polynomial $p(x)$ which satisfies the conditions in the previous problem and for which we have $p^{(k+1)}(x)$ is a non-zero constant.

October 22, 2004