MA/CS 321:001
MWF 11:00–11:50
FB 213
Fall 2004

Instructor: Russell Brown
Office: POT741
Phone: 257-3951
russell.brown@uky.edu

Announcements.

1. As many of you already know, matlab is available in the Cisco Systems lab in the basement of Young Library. On my way back from Young, I dropped by the Chemistry-Physics computer lab and found it available there, also. You appear to be able to log in with your u-connect account. It is amusing to note that Young has version 6.5 and Chemistry-Physics has version 7.0 of matlab.

2. I plan to hand out solutions to Homework 4 in class on Monday and will have the papers graded by Tuesday morning at 10am. They will be available in an envelope taped to my office door. I encourage you to collect these papers before the exam.

3. The first exam will be in class on Wednesday, 29 September 2004. This exam will cover the material we have discussed. Taylor's theorem, floating point arithmetic, loss of precision, the bisection method and Newton's method.

Sample questions.

In answering these questions, you should try to explain the reasoning you used as well as the answer.

1. State Taylor's theorem with remainder.

2. State the theorem on loss of precision.

3. Write out the Taylor polynomial of degree $2n+1$ centered at zero for $\sin x$. What degree Taylor polynomial do we need to compute $\sin(2)$ with an error of no more than 10^{-3}.

4. If we are trying to compute x and we know that $y = 5$ approximates x with a relative error of 0.1. What are the largest and smallest possible values of x?

5. If we are trying to compute x and we know that $y = 0$ approximates x with a relative error of 0.001, what are the largest and smallest possible values of x?

6. Suppose that we have binary floating point numbers of the form $(-1)^s(1.f)_2 \times 2^{m-512}$. Here s is a one bit sign, f is a twenty bit mantissa and m is 10 bit biased exponent.

   For these numbers answer the following questions.

   (a) What is the largest number that can be represented this way?

(b) If x is the smallest machine number which is larger than 15, what is the value of $x - 15$?

(c) What is the output of the following matlab function, if it were run on a machine using the arithmetic described above.

```
function u = epsilon()
  u = 1;
  while (2*u+ 16 > 16)
   u = u/2;
  end
```

(d) Suppose that we wish to run the bisection method with a starting interval of $1/10, 1/9$. How many steps should we use to compute the answer to machine accuracy?

(e) If x is a number which is about 29, what is the error committed in rounding to the nearest machine number?

(f) If x and y are near 29 and 63, respectively, what is the relative error in computing $x + y$ on this machine?

7. Suppose we are working with floating point decimal arithmetic and each arithmetic operation is rounded to 6 significant figures. If we use the standard quadratic formula to compute the roots to the equation $x^2 + 10^4 x - 1 = 0$, what are the computed roots?

   One of the roots is computed with a larger relative error than the other. Which one is it? Suggest a better method for computing this root.

8. We are to compute $e^x - e^{-x}$ in finite precision arithmetic and are allowed to lose at most 2 binary digits of precision. Use the theorem on loss of significance to find the values of x for which this is possible.

9. Write a function $f(x)$ which has $\sqrt[3]{2}$ as a root. Using $x_0 = 2$ as a starting guess, find x_1 and x_2.

   Your iteration formula should only require addition, subtraction, multiplication and division.

10. Suppose f is a function with $f(r) = 0$, $|f''(x)| \leq 10$ and $|f'(x)| \geq 13$. If x_n and x_{n+1} are consecutive iterates of Newton's method, then

$$|x_{n+1} - r| \leq M|x_n - r|^2.$$

    Prove this and give an explicit value for M.