MA/CS 321:001
MWF 11:00–11:50
FB 213
Fall 2004

Instructor: Russell Brown
Office: POT741
Phone: 257-3951
russell.brown@uky.edu

Topics covered.

- Horner's method.
- Polynomial interpolation. Lagrange form, Newton form, uniqueness and divided differences.
- Error estimates for polynomial interpolation. Proof and applications. You do not need to memorize the statements of the theorems I and II on error in polynomial interpolation. You should be familiar with the steps in the proof.
- Computing derivatives. Using extrapolation to improve error bounds.
- Simpson's rule, trapezoid rule. Error estimates. You do not need to memorize the error estimates.

Some suggestions.

- Study your lecture notes.
- Study your homework and solutions.
- Be sure that you understand the reasoning used.
- If needed, the four theorems on error, two for polynomial interpolation, one for the trapezoid rule and one for Simpson's rule, will be provided for you on the exam.
- You should know the basics of Taylor series and computer arithmetic from the first part of the course.

Sample problems

1. Give the value that `ff(2)` returns where `ff` is the matlab function:

```
function u = ff(x)

u = 0;

for k=1:4
  u = (k+1) + x*u
end
```

2. Write out the Lagrange form of the interpolating polynomial, p_2 for the values $f(1) = 2$, $f(3) = 0$ and $f(5) = 0$.

   Find $\int_1^5 p_2(x)\, dx$.

3. Suppose that we use an interpolating polynomial of degree n, $p_n(x)$ with equally spaced nodes to approximate $\cos(3x)$ on $[0, 1]$. How many nodes do we need to ensure that $|\cos(x) - p_n(x)| \leq 10^{-3}$?

4. Suppose that all derivatives of f exist on the interval $[a, b]$. If $f(a) = f'(a) = f(b) = f'(b) = 0$, find the largest index k for which we can prove that there is a ξ in (a, b) so that $f^{(k)}(\xi) = 0$.

   Let k be the index you found above. Can you find an example of a function which satisfies the conditions in the first part of this problem and where $f^{(k+1)}(\xi)$ is not zero? Hint: Try a polynomial on $[0, 1]$ where the $k + 1$st derivative is constant.

5. Can you find a polynomial which satisfies $p(0) = 1$, $p'(0) = 2$ and $p(1) = 3$?

6. Find the polynomial of lowest degree which satisfies $p(0) = 1$, $p(1) = 2$, $p(3) = 4$ and $p(5) = 6$.

7. Complete the divided difference table and write out the Newton form of the interpolating polynomial for the following table.

k	x_k	y_k
0	1	2
1	2	5
2	4	17
3	5	26

8. State the uniqueness theorem for polynomial interpolation.

9. Find a value of α so that
$$\frac{f(x+h) - f(x-2h)}{h}$$
   approximates $\alpha f'(x)$ as $h \to 0$. Suppose that all derivatives of f exist. Find an expression for the error.

10. Can you find α and β so that
$$\frac{\alpha f(x+3h) + \beta f(x+h) - f(x-2h)}{4h} = f'(x) + O(h)?$$

    Hint: Expand everything on the left in a Taylor series.

11. Suppose that

$$\phi(h) = L + \sum_{j=1}^{\infty} a_j h^{3j}.$$

Can you find a combination of $\phi(h)$ and $\phi(-h)$ which approximates L with an error term of the form $O(h^6)$?

Can you find a combination of $\phi(h)$ and $\phi(h/2)$ which approximates L with an error term of the form $O(h^6)$?

12. Use Simpson's rule with $n = 4$ to approximate

$$\int_1^2 \cos(x)\, dx.$$

13. Criticize the following argument.

Let f be a differentiable function on $[-1, 1]$. By the mean-value theorem, $f(x) = f(0) + f'(\xi)x$. Thus, we can integrate and obtain that

$$\int_{-1}^{1} f(x)\, dx = \int_{-1}^{1} f(0)\, dx + f'(\xi) \int_{-1}^{1} x\, dx = 2f(0).$$

14. How many subintervals do we need to use the trapezoid rule to approximate

$$\int_1^2 \frac{1}{x}\, dx$$

with an error of at most 10^{-3}.

15. Answer the previous question for Simpson's rule.

October 29, 2004