Introduction to Partial Differential Equations
MWF 12–12:50pm
CB343
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and by appointment.

The midterm exam will ask you to solve 3 of the following problems. Be prepared.

1. Find the solution of the problem

$$\begin{cases} u_t + u_x = u, & \text{in } \mathbf{R}^2 \\ u(x,0) = \sin(x), \qquad x \in \mathbf{R}. & \end{cases}$$

2. Find $\delta > 0$ so that A defined by

$$Af(t) = \int_0^t \sin(f(s))\,ds$$

is a contraction on $C([-\delta,\delta])$ with the metric

$$\|f - g\| = \max_{t\in[-\delta,\delta]} |f(t) - g(t)|.$$

3. Prove that if u is in $C^2(\Omega)$,

$$\phi(r) = \fint_{\partial B(x,r)} u(y)\,dy,$$

then whenever the closed ball $\bar{B(x,r)} \subset \Omega$, then

$$\phi'(r) = \frac{r}{n} \fint_{B(x,r)} \Delta u(y)\,dy.$$

4. Prove Green's second identity. If u, v are $C^2_c(\mathbf{R}^n)$, then

$$\int u(x)\Delta v(x) - v(x)\Delta u(x)\,dx = 0.$$

5. If A is an $n \times n$ matrix and

$$(\Delta u)(Ax) = \Delta(u \circ A)(x)$$

for all u which C^2 in $\mathbf{R}^n$, then A is orthogonal.

Hint: It suffices to consider the functions $u_{ij}(x) = x_i x_j$.

6. Use the mean-value theorem for harmonic functions to prove the maximum principle for harmonic functions.

7. If u is $C_c^2(\mathbf{R}^n)$, show that

$$\int_{\mathbf{R}^n} \Phi(x-y)\Delta u(y)\,dy = -u(x).$$

Here, Φ is the fundamental solution for the Laplacian.

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