Introduction to Partial Differential Equations
MWF 12–12:50pm
CB343
Fall 2005

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and by appointment.

The due date on homework 1 is now Monday, 12 September 2005.
Homework 3. Due Monday, 26 September 2005.

1. Evans, p. 85 #1.

2. Evans, p. 85 #2.

3. This problem outlines the construction of radial solutions to $\Delta u + u = 0$ in $\mathbf{R}^3$.

   (a) If $u(x) = f(|x|)$ and $u$ solves $\Delta u + u = 0$, find a second order ordinary differential equation that $f$ must solve.

   (b) Look for solutions of the ordinary differential equation you found in part a) by writing $f(r) = \sum_{j=-1}^{\infty} a_j r^j$.
   Formally differentiate this series term by term and substitute the series for $f$, $f'$ and $f''$ into the ordinary differential equation.
   Collect powers of $r$ and find a recurrence relation for the coefficients $a_j$.

   (c) Find two linearly independent solutions of this ordinary differential equation. For the first one solve the recurrence relation with the initial conditions $a_{-1} = 0$ and $a_0 = 1$. For the second solution, use the initial condition $a_{-1} = 1$ and $a_0 = 0$.

   (d) can you identify the series that you obtained in part c)? Hint: Look up the series for $\sin(x)$ and $\cos(x)$ if you have not taught Calculus 2 recently. Only this step requires that $n = 3$. In other dimensions, the corresponding solutions involve Bessel functions.

4. Suppose that we are in two dimensions. If $u$ is a function on $\mathbf{R}^2$, define a function $v$ by $v(r, \theta) = u(r\cos\theta, r\sin\theta)$. Express $\Delta u$ in terms of derivatives with respect to $r$ and $\theta$. That is, find a differential operator $L$ involving $\frac{\partial}{\partial r}$ and $\frac{\partial}{\partial \theta}$ so that
$$(\Delta u)(r\cos\theta, r\sin\theta) = Lv(r, \theta).$$

September 26, 2005