Introduction to Partial Differential Equations
MWF 12–12:50pm
CB343
Fall 2005

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Homework 4. Due Wednesday, 5 October 2005.

1. Evans, p. 85 #3
2. Evans, p. 85 #4
3. The operator $\bar{\partial}$ in two dimensions is defined by

$$\bar{\partial} = \frac{1}{2}\left(\frac{\partial}{\partial x} + i\frac{\partial}{\partial y}\right).$$

   (a) Let $z = x + iy$ and show $\bar{\partial}\bar{z} = 1$ and $\bar{\partial}z = 0$.

   (b) Let $\Phi(x, y) = 1/(\pi z) = 1/(\pi(x + iy))$. Show that $\bar{\partial}\Phi = 0$ if $(x, y) \neq 0$.

   (c) Given f a $C^1_c(\mathbf{R}^2)$ function, define u by

$$u(x, y) = \int_{\mathbf{R}^2} \Phi(x - s, y - t)f(s, t)\, dsdt.$$

   Show that u is in $C^1(\mathbf{R}^2)$ and $\bar{\partial}u = f$. Hint: You will want to imitate the proof that $-\Delta u = f$ when u is the Newtonian potential. In place of Green's second identity, you will need to use the divergence theorem to write

$$\int_\Omega \frac{\partial u}{\partial x}\Phi + u\frac{\partial \Phi}{\partial x}\, dxdy = \int_{\partial\Omega} u\Phi\nu \cdot e_1\, d\sigma$$

   and a similar formula for the y-derivatives.

September 28, 2005