Introduction to Partial Differential Equations
MWF 12–12:50pm
CB343
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Homework 5. Due Wednesday, 19 October 2005.

1. Evans, p. 85 #5. Hint: Find a constant M so that $\pm u(x) + M(1 - |x|^2)$ is subharmonic. Use the maximum principle.

2. Evans, p. 85 #8. For simplicity, you may assume that $g(x) = |x|$ if $|x| \leq 1$ and $g(x) = 0$ for $|x| > 1$. However, as Evans indicates, as long as g is bounded, the conclusion is independent of the values of g outside a neighborhood of the origin.

   This problem is interesting because it shows that if g is Lipschitz, we do not necessarily have that u is Lipschitz. There is a class called the Zygmund class which is slightly larger than the class of Lipschitz functions and so that if g is in the Zygmund class, then u is in the Zygmund class. This helps to explain why we have a sawtooth metal strip on every package of Saran wrap.[1]

3. Suppose that A is a linear transformation and $\Delta(u \circ A) = (\Delta u) \circ A$ for all C^2 functions u. Show that A is orthogonal. Hint: The functions $x_i x_j$ might be useful.

October 10, 2005

[1] I have heard that this last observation is due to Zygmund.