Introduction to Partial Differential Equations
11-11:50am
Main 0003
Fall 2012

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Homework 3. Due Monday, 24 September 2012

1. Evans, p. 85, # 10
2. Evans, p. 85, # 11
3. The differential operator $\bar{\partial}$ in two dimensions is defined by
$$\bar{\partial} = \frac{1}{2}\left(\frac{\partial}{\partial x} + i\frac{\partial}{\partial y}\right).$$
   (a) Let $z = x + iy$ and show $\bar{\partial}\bar{z} = 1$ and $\bar{\partial}z = 0$.
   (b) Let $\Phi(x,y) = 1/(\pi z) = 1/(\pi(x+iy))$. Show that $\bar{\partial}\Phi = 0$ if $(x,y) \neq 0$.
   (c) Given f a $C^1_c(\mathbf{R}^2)$ function, define u by
$$u(x,y) = \int_{\mathbf{R}^2} \Phi(x-s, y-t) f(s,t)\, dsdt.$$
   Show that u is in $C^1(\mathbf{R}^2)$ and $\bar{\partial}u = f$. Hint: You will want to imitate the proof that $-\Delta u = f$ when u is the Newtonian potential. In place of Green's second identity, you will need to use the divergence theorem to write
$$\int_\Omega \frac{\partial u}{\partial x}\Phi + u\frac{\partial \Phi}{\partial x}\, dxdy = \int_{\partial\Omega} u\Phi\nu \cdot e_1\, d\sigma$$
   and a similar formula for the y-derivatives.
4. Suppose that we are in two dimensions. If u is a function on $\mathbf{R}^2$, define a function v by $v(r,\theta) = u(r\cos\theta, r\sin\theta)$. Express Δu in terms of derivatives with respect to r and θ. That is, find a differential operator L involving $\frac{\partial}{\partial r}$ and $\frac{\partial}{\partial \theta}$ so that
$$(\Delta u)(r\cos\theta, r\sin\theta) = Lv(r,\theta).$$
5. No recitation on 14 September. Contrary to my earlier announcement, I would like to have recitation on 21 September.

September 12, 2012