Introduction to Partial Differential Equations 11-11:50am Main 0003 Fall 2012 Instructor: Russell Brown Office: POT741 Phone: 257-3951 russell.brown@uky.edu Office Hours: WF 2-3 in POT 741 and by appointment.

Homework 4. Due Wednesday, 3 October 2012

1. Let ϕ be defined by

$$\phi(t) = \begin{cases} \exp(-1/t), & t > 0\\ 0, & t \le 0 \end{cases}$$

Show that ϕ is in $C^{\infty}(\mathbf{R})$. One way to do this is to show by induction that $\phi^{(k)}(t) = P_k(1/t) \exp(-1/t)$ for t > 0 where P_k is a polynomial. What is the degree of this polynomial?

Show that there is a nonzero function in $C_c^{\infty}(\mathbf{R})$.

- 2. Evans, p. 85, # 12
- 3. Evans, p. 85, # 14 Hint: If u is a solution of the heat equation, what equation does $\exp(ct)u(x,t)$ solve?

October 2, 2012