Introduction to Partial Differential Equations
11-11:50am
Main 0003
Fall 2012

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and by appointment.

Homework 5. Due Friday, 12 October 2012
Please remember that you are to attend 4 seminars or other events and hand in written summaries.

1. Suppose that f is in $C^1(\mathbf{R})$ and f is an even function. What can you say about f' and $f'(0)$?

2. (a) Show that if g is a bounded, continuous function on $\mathbf{R}^n$ which is even in the x_n variable, then the solution of

$$\begin{cases} D_t u - \Delta u = 0 & (x,t) \in \mathbf{R}^n \times (0,\infty) \\ u(x,0) = g(x), & x \in \mathbf{R}^n \end{cases}$$

   is also even.

   (b) Let g be bounded and continuous on $\mathbf{R}^n_+ = \{(x', x_n) : x_n \geq 0\}$.
   Find an explicit formula for the solution of the problem

$$\begin{cases} D_t u - \Delta u = 0 & (x,t) \in \mathbf{R}^n_+ \times (0,\infty) \\ u(x,0) = g(x), & x \in \mathbf{R}^n_+ \\ \frac{\partial u}{\partial \nu}(x,t) = 0, & (x,t) \in \partial\mathbf{R}^n_+ \times (0,\infty). \end{cases}$$

   Here $\partial/\partial\nu$ is the derivative in the direction of the outward unit normal to $\{x : x_n > 0\}$. Of course this is just $-\partial/\partial x_n$.

3. Write a problem similar to #2 which considers functions g which are odd. You do not need to give a solution.

4. This problem is an extension of the maximum principle for Laplace's equation. But the argument is similar to our proof of the maximum principle for the heat equation in $\mathbf{R}^n \times (0,T)$.

   Let $n \geq 3$. Suppose that $\Delta u = 0$ in $\{x : 0 < |x| < 1\}$ and that u is continuous in $\{x : 0 < |x| \leq 1\}$. Assume that there are constants $\epsilon > 0$ and C so that $u(x) \leq C|x|^{2-n+\epsilon}$.

   Show that

$$\sup_{\{x:0<|x|\leq 1\}} u(x) \leq \max_{\{x:|x|=1\}} u(x)$$

   Hint: Remember that $|x|^{2-n}$ is a solution of Laplace's equation if $x \neq 0$.

5. Suppose that u is a solution of Laplace's equation in the punctured ball $\{x : 0 < |x| < 1\}$, that u is continuous in $\{x : 0 < |x| \leq\}$, and for some C and $\epsilon > 0$, $|u(x)| \leq C|x|^{2-n+\epsilon}$. Show that u has a continuous extension to the unit ball and that this extension is harmonic in the open unit ball.

   Hint: Poisson integral

6. #16 in Evans, page 88.

October 2, 2012