Read Chapter 1 and 2 of Beals. Work the following exercises. Hand in solutions on Friday, 1 September.

1. (Beals, page 9) Prove that the equation $r^2 = 3$ does not have a rational solution.

2. (Beals, page 14) Let a and b be two real numbers. Show that we have $a \leq b$ if and only if for each $\epsilon > 0$ we have $a < b + \epsilon$. Hint: Trichotomy.

3. Prove that for all real numbers a and b, we have $ab \leq \frac{1}{2}(a^2 + b^2)$. Hint: First show that if x is real, then $x^2 \geq 0$.

4. Prove that for all real numbers a and b we have $a^3 b^3 \leq \frac{1}{4}a^{12} + \frac{3}{4}b^4$.

5. Show that a bounded non-empty set of real numbers has at most one least upper bound.