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Due Wednesday, 13 September 2006.

1. (Beals, page 20) For two sets A and B, we define $A+B=\{c : c=a+b \text{ with } a\in A,\ b\in B\}$. Show that $\sup A+B=\sup A+\sup B$.
2. Can you show that if E is a set of real numbers, then $\sup E\in E$?
3. (Beals, page 20) Show that, for any positive reals x and y, there is a positive integer n so that $x/n<y$.
4. (Beals, page 20) *The nested interval property.* Suppose that $I_1,\ I_2,\dots$ is a sequence of bounded closed intervals with $I_1\subset I_2\subset I_3\dots$. Let $I_n=[a_n,b_n]$ and suppose that $\lim_{n\to\infty} b_n-a_n=0$.

   Show that $\cap_{n=1}^{\infty} I_n$ is not empty. Is the result still true if the intervals are not bounded?

   Hint: What can you say about $\sup\{a_n : n=1,2,\dots\}$.
5. (Beals, page 20) Prove that for any positive h and and any integer $n\geq 0$, we have

   $$(1+h)^n \geq 1+nh+\frac{n(n-1)}{2}h^2.$$

   Hint: Binomial theorem. You may assume the binomial theorem.
6. (Beals, page 20) Suppose that a is positive and $n\geq 2$ is an integer. Suppose that $a^n=n$. Prove that $1<a<1+\sqrt{2/(n-1)}$.
7. Prove that $\lim_{n\to\infty} n^{1/n}=1$.
8. Negate the definition of a sequence which converges to a. That is write out what it means for a sequence $\{a_n\}$ to not converge to a.
9. (Beals, page 29) Prove that there is no way to put an order relation on $\mathbf{C}$ so that $\mathbf{C}$ is an ordered field. An ordered field satisfies the axioms A1-4, M1-4, D and O1-4.
10. (Beals, page 29) Show that if a, b and c are complex numbers, $|a|=|b|=|c|$ and $a+b+c=0$, then $|a-b|=|b-c|=|a-c|$. Discuss the geometric significance.

Additional questions.(Not to be collected.)

1. Is π^e transcendental?