MA575 Instructor: Russell Brown
MWF 9-9:50pm Office: POT741
CB 343 Phone: 859 257 3951
Fall 2006 russell.brown@uky.edu

Due Thursday, 5 October 2006.
Our exam will be on 13 October 2006. The examination will cover through Chapter 4/Lecture 16.
For the exam, you should be prepared to state a definition, state and prove a theorem, solve a problem and give an example.

1. Find $l.u.b.\,\mathbf{R}$ and $l.u.b.\,\emptyset$.

2. Omitted.

3. (Beals, page 44) The Fibonacci sequence is the sequence $\{F_n\}$ defined by $F_1 = F_2 = 1$ and
$$F_{n+2} = F_{n+1} + F_n \qquad (1)$$
for $N = 3, 4, \ldots$.
   (a) Find values of r so that the sequence $F_n = r^n$ satisfies (1).
   (b) Suppose that x_n and y_n satisfy (1) and a and b are complex numbers. Show that $z_n = ax_n + by_n$ satisfies (1).
   (c) Find a, b, r, and s so that
$$F_n = ar^n + bs^n, \qquad n = 1, 2, \ldots.$$

4. (Beals, p. 48) Suppose that a series $\sum_{n=1}^{\infty} x_n$ has $x_1 \geq x_2 \geq x_3 \ldots$ and $x_n \geq 0$ for all n. Show that if $\sum x_n$ converges, then $\lim_{n\to\infty} nx_n = 0$.

5. Suppose that $\{x_n\}_1^\infty$ is a sequence with positive terms. Show that $\lim_{n\to\infty} x_n = \infty$ if and only if $\lim_{n\to\infty} 1/x_n = 0$.

6. (Beals, page 53) Determine if the following series converge. You may use the integral test, if necessary.
   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n n!}{n^n}$
   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{1\cdot 3\cdot 5\ldots(2n-1)}{2\cdot 4\cdot 6\ldots(2n)}$
   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{2\cdot 4\cdot 6\ldots(2n)}{1\cdot 3\cdot 5\ldots(2n+1)}$

(d) $\sum_{n=1}^{\infty}(n^{1/n}-1)$

Additional questions

1. (Beals) Suppose that $\{x_n\}_{n=1}^{\infty}$ is a bounded real sequence and $b = \limsup_{n\to\infty} x_n$. Show that there is a monotone sequence converging to b.

2. Find the value of the series $\sum_{n=1}^{\infty} \frac{1}{n^3}$.

3. This problem gives another way to define the sum of a series with non-negative terms. Consider a series Σx_n with $s = \Sigma_{n=1}^{\infty} x_n$. For a finite set $A \subset \mathbf{N}$ we define the sum $\sigma_A = \sum_{n\in A} x_n$. Then, we put $\sigma = l.u.b.\{\sigma_A : A \text{ is a finite subset of } \mathbf{N}\}$.

   If $\Sigma_{n=1}^{\infty} x_n$ is a series with $x_n \geq 0$ show that the series converges if and only if σ is finite. If the case that the series converges, show that $s = \sigma$.

4. Let $\{x_n\}$ and $\{y_n\}$ be bounded sequences and suppose that $x_n \leq y_n$ for every n. Can you show that
$$\limsup x_n \leq \liminf y_n?$$