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Homework 7. Due Friday, 17 November 2006.

1. Let X and Y be metric spaces and $f : X \to Y$ be a function. Show that the limit of f at a point x is unique. In other words, show that if $\lim_{y\to x} f(y) = L$ and $\lim_{y\to x} f(y) = M$, then $L = M$. The solution to this problem should help to explain why we take limits at limit points.
2. Suppose that $f : X \to Y$ is a function.
   (a) Show that for $B \subset Y$, we have $f(f^{-1}(B)) \subset B$.
   (b) Give an example to show that we may have strict containment in part (a).
   (c) A function $f : X \to Y$ is *surjective* or onto if for each $y \in Y$, there is an $x \in X$ so that $f(x) = y$. Show that f is surjective if and only if $f(f^{-1}(B)) = B$ for all sets $B \subset Y$.
3. Suppose that $f : X \to Y$ is a function.
   (a) Show that for $A \subset X$, we have $A \subset f^{-1}(f(A))$.
   (b) Give an example to show that we may have strict containment in part (a).
   (c) A function f is *injective* or one-to-one if $f(x) = f(x')$ implies that $x = x'$. Show that f is injective if and only if $A = f^{-1}(f(A))$ for all sets $A \subset X$.
4. If $f : X \to Y$ is continuous, is $f^{-1}(F)$ closed whenever $F \subset Y$ is closed?
5. Suppose that X and Y are metric spaces and $f : X \to Y$ is a uniformly continuous function. Show that if $\{x_n\}_{n=1}^{\infty}$ is a Cauchy sequence in X, then $\{f(x_n)\}_{n=1}^{\infty}$ is a Cauchy sequence in Y.
6. A set A is *dense* in a metric space X if $cl(A) = X$. Suppose that A is a dense subset of a metric space X and $f : X \to \mathbf{R}$ is a continuous function. Show that if $g : X \to \mathbf{R}$ is continuous and $g(x) = f(x)$ for all $x \in A$, then $g = f$.
7. Suppose that A is a dense subset of a metric space X and that $f : A \to \mathbf{R}$ is uniformly continuous. Show that there is a (unique) function $g : X \to \mathbf{R}$ which is continuous and satisfies $g(x) = f(x)$ for $x \in A$.

Additional problem.

1. Evaluate the integral,

$$\int_0^\infty (x - \frac{x^3}{2} + \frac{x^5}{4\cdot 2} - \frac{x^7}{6\cdot 4\cdot 2} + \ldots)(1 + \frac{x^2}{2^2} + \frac{x^4}{2^2\cdot 4^2} + \frac{x^6}{2^2\cdot 4^2\cdot 6^2} + \ldots)\, dx$$

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