MA575 Instructor: Russell Brown
MWF 9-9:50pm Office: POT741
CB 343 Phone: 859 257 3951
Fall 2006 russell.brown@uky.edu

To be discussed during dead week.

1. Let $f_n(x) = \frac{nx}{1+n^2x}$. Show that
$$\lim_{n\to\infty} f_n(x) = 0.$$
Does the sequence $f_n(x)$ converge uniformly?

2. Let
$$f(x) = \begin{cases} 1, & x \geq 0 \\ 0, & x < 0 \end{cases}$$
Show that f is Riemann integrable on $[-1, 1]$.

3. (Beals p. 112)
   (a) Suppose that f is a continuous real-valued function on the interval $[a, b]$ and that $\alpha : [a, b] \to \mathbf{R}$ is non-decreasing. Prove that there exists $c \in [a, b]$ so that
   $$\int_a^b f\, d\alpha = f(c)(\alpha(b) - \alpha(a)).$$
   (b) Give another proof when $\alpha(t) = t$. (Hint: One proof might use the intermediate value theorem. The other proof should use the mean-value theorem.)

4. If $\{f_n\}$ is a sequence of Riemann integrable functions on an interval $[a, b]$ and f_n converges uniformly to f, show that f is Riemann integrable and that
$$\lim_{n\to\infty} \int_a^b f_n\, dx = \int_a^b f\, dx.$$

5. Suppose that $f_n : [0, 1] \to \mathbf{R}$ is a sequence of continuous functions and $\lim_{n\to\infty} f_n(x) = 0$ for each $x \in [0, 1]$. Do we have
$$\lim_{n\to\infty} \int_0^1 f_n\, dx = 0?$$

6. Suppose that $f : [0, 1] \to [0, \infty)$ is continous and
$$\int_0^1 f dx = 0.$$
Does this imply $f = 0$? Can we conclude that $f = 0$ if f is only Riemann integrable?

Additional problems.

1. Let C be the Cantor set. Define a function $f : [0, 1] \to \mathbf{R}$ by

$$f(x) = \begin{cases} 1, & x \in C \\ 0, & x \notin C \end{cases}$$

Show that the function f is Riemann integrable on the interval $[0, 1]$.

November 30, 2006