Theory of partial differential equations
MWF 9-9:50am
CB343
Spring 2013

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Homework 2, due on Wednesday, 30 January 2013

1. Let $\phi : \mathbf{R}^{n-1} \to \mathbf{R}$ be a Lipschitz function with Lipschitz constant M. Thus,

$$|\phi(x') - \phi(y')| \leq M|x' - y'|.$$

   Let $U = \{(x', x_n) : x_n > \phi(x')\}$ and suppose that $x \in U$. Show that $\mathrm{dist}(x + te_n, \partial U) > ct$. Express c in terms of M. Hint: Consider the special case when $\phi(x') = M|x'|$ in order to find a candidate for c.

   Note that this is essentially the technical fact used in the proof our global approximation theorem, Theorem 5 of Lecture 4.

2. Prove the following form of the product rule. Suppose that u and $Du = (D_{x_1}u, \ldots, D_{x_n}u)$ are in $L^p_{loc}(U)$ and v and Dv are $L^q_{loc}(U)$ with $1/p + 1/q = 1$. Prove that uv and $D(uv)$ are in $L^1_{loc}(U)$ and that $D(uv) = uDv + vDu$. Hint: First consider the case when one of the functions is in $C^\infty(U)$. Be careful if one of the indices p or q is infinite.

3. Evans p. 306, #7.

4. Evans p. 307, #8.

18-25 January, reading

- §5.3, Approximations
- §5.4, Extensions
- §5.5, Traces

January 23, 2013