Theory of partial differential equations
MWF 9-9:50am
CB343
Spring 2013

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Homework 6, due on 29 March 2013

1. Let $J : W_0^{1,2}(U) \to W^{-1,2}(U)$ be the map used in the proof of Theorem 2, Lecture 17 and defined by

$$\langle J(u), v\rangle = \int_U uv\,dx.$$

The adjoint is naturally defined as a map $J^* : W^{-1,2}(U)^* \to W^{-1,2}(U)$. But if we let $I : W_0^{1,2}(U) \to W^{-1,2}(U)^*$ be the isomorphism given by $I(u)(v) = \langle v, u\rangle$, we may consider $J^* \circ I : W_0^{1,2}(\Omega) \to W^{-1,2}(\Omega)$. Give a simple description of the map $J^* \circ I$.

Hint: This is really easy, once you figure out what the problem asks.

2. Evans #4. Hint: Consider the form $B(u,v) = \int_U \nabla u \cdot \nabla v\,dx$. This form is not coercive on $W^{1,2}(U)$. However, if ∂U is C^1, then the form is coercive on the subspace $W^{1,2}(U) \cap \{u : \int u\,dx = 0\}$. Why? Note the standing assumption that ∂U is smooth which appears at the beginning of the problem section of Chapter 6.

3. Evans, #6.

- Read section 6.3 on regularity for weak solutions of elliptic equations.

March 20, 2013