

MA676
MWF 2-2:50pm
CB 347
Spring 2007

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EXERCISES

1. Find $\limsup_{k \rightarrow \infty} E_k$ and $\liminf_{k \rightarrow \infty} E_k$ if

$$E_k = \begin{cases} [-1/k, 1], & k \text{ odd} \\ [-1, 1/k], & k \text{ even} \end{cases}$$

2. State deMorgan's laws relating unions, intersections and complements of sets.

Show that

$$\limsup_{k \rightarrow \infty} (\mathbf{R}^n \setminus E_k) = \mathbf{R}^n \setminus \liminf_{k \rightarrow \infty} E_k$$

3. Show that $\limsup_{k \rightarrow \infty} E_k$ is the set of points which lie in infinitely many of the sets E_k .

State and prove a similar characterization for $\liminf_{k \rightarrow \infty} E_k$.

4. Let $x_j \in \mathbf{R}$ for $j = 1, \dots, n$. Show that

$$\left(\sum_{k=1}^n x_k \right)^2 \leq n \sum_{k=1}^n x_k^2.$$

Give necessary and sufficient conditions on the numbers x_k which guarantee that we have equality in this inequality.

PROBLEMS

Due 17 January 2007.

1. Prove part a) of Theorem 1.4 in Wheeden and Zygmund.
2. Let a_k and b_k be sequences in the extended reals. If both $\limsup_{k \rightarrow \infty} a_k$ and $\limsup_{k \rightarrow \infty} b_k$ are infinite, assume that both have the same sign.

Show that

$$\limsup_{k \rightarrow \infty} (a_k + b_k) \leq \limsup_{k \rightarrow \infty} a_k + \limsup_{k \rightarrow \infty} b_k.$$

Give an example to show that we may have strict equality.

Remark: This exercise is familiar from last semester. In your explanation, you should strive to make it clear that our arguments continue to work when one or both of the limits are infinite.

3. Find $\limsup_{n \rightarrow \infty} \sin(n)$.

January 9, 2007