MA676 Instructor: Russell Brown
MWF 2-2:50pm Office: POT741
CB 347 Phone: 859 257 3951
Spring 2007 russell.brown@uky.edu

Exercise set 10.

1. If f is in $L^1(E)$, can you find
$$\lim_{\lambda\to\infty}\int_{\{|f|>\lambda\}} |f|\,dx?$$

2. If f is in $L^1(E)$, can you find
$$\lim_{\lambda\to\infty}\lambda|\{|f|>\lambda\}|?$$

3. If f is in $L^1(\mathbf{R})$, can you find
$$\lim_{\lambda\to 0}\int_{\mathbf{R}} f(x)\sin(\lambda x)\,dx?$$

4. (Wheeden and Zygmund) If f is a real-valued function defined on $\mathbf{R}^n$ and we have that
$$\int_A f\,dx = 0$$
over every measurable subset A of $\mathbf{R}^n$, show that $f = 0$ a.e.

5. In $\mathbf{R}^n$, show that $|B(x,r)| = |B(0,1)|r^n$.

6. In $\mathbf{R}^n$, find the values of α for which we have
$$\int_{B(0,1)} |x|^{-\alpha}\,dx < \infty.$$
Hint: Consider integrals over the sets $B(0,2^{-k})\setminus B(0,2^{-k-1})$.

Problem set 10.

These problems should be handed in on Friday, 7 April 2007.

1. If $\{f_k\}$ is a sequence of functions in $L^1(\mathbf{R})$ and there is a measurable function f so that
$$\lim_{k\to\infty}\int |f_k - f|\,dx = 0.$$
Show that $\{f_k\}$ converges in measure to f. Hence, it is easy to conclude that a subsequence, $\{f_{k_j}\}$ converges a.e.

2. If $f\in L^1([0,1])$, find
$$\lim_{k\to\infty}\int_0^1 x^k f(x)\,dx.$$

March 30, 2007