MA676 MWF 2-2:50pm CB 347 Spring 2007 Instructor: Russell Brown Office: POT741 Phone: 859 257 3951 russell.brown@uky.edu

## Exercise set 11.

- 1. For which values of  $\alpha$  is  $t^{\alpha}$  in  $L^1([0,1]), L^1([1,\infty))$  or  $L^1([0,\infty)?$
- 2. Show that

$$\lim_{R \to \infty} \int_0^R \frac{\sin(x)}{x} \, dx$$

exists. Hint: Integrate by parts on the interval [1, R].

- 3. Is  $\sin(x)/x$  in  $L^1([0,\infty))$ ?
- 4. (Hard?) If  $f \in L^1(\mathbf{R})$  and  $\int_Q f(x) dx = 0$  for every cube Q in  $\mathbf{R}^n$ , is f = 0 a.e.?

## Problem set 11.

These problems should be handed in on Friday, 13 April 2007.

- 1. For which values of s is  $e^{-t}t^{s-1}$  in  $L^1((0,\infty))$ ?
- 2. The gamma function is defined for s > 0 by

$$\Gamma(s) = \int_0^\infty e^{-t} t^s \, \frac{dt}{t}.$$

Show that

$$\Gamma(s+1) = s\Gamma(s).$$

Hint: Integrate by parts. Conclude that  $\Gamma(n+1) = n!$  for n = 0, 1, 2, ...

April 5, 2007