MA676
MWF 2-2:50pm
CB 347
Spring 2007

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Announcement

This will be our last graded homework set.

Exercise set 12.

1. For which values of α is $|x|^{-1}(-\log(|x|)^{\alpha}$ in $L^1((0,1/2))$?

2. Let $f(x) = \chi_{((0,1/2))}(x)|x|^{-1}(\log(|x|))^{-2}$. Show that there is a constant $c > 0$ so that

$$f^*(x) \geq \frac{c}{|x|(-\log(|x|))}, \qquad \text{if } |x| < 1/2.$$

   Remark: This exercise shows that there is a function f in L^1 for which f^* is not locally integrable.

Problem set 12.

These problems should be handed in on Friday, 20 April 2007.

1. (a) Show that

$$\int_{\mathbf{R}} \cos(tx)\exp(-x^2)\,dx = \sum_{k=0}^{\infty} \int_{\mathbf{R}} \frac{(-1)^k (tx)^{2k}}{(2k)!}\exp(-x^2)\,dx.$$

   Hint: Use that $\cosh(x) = \sum_{k=0}^{\infty} \frac{x^{2k}}{(2k)!}$ to help find a dominating function.

   (b) Express

$$\int_{\mathbf{R}} x^{2k}\exp(-x^2)\,dx$$

   in terms of $\Gamma(1/2)$ where $\Gamma(s)$ is the function introduced in an earlier homework set. It is known that $\Gamma(1/2) = \sqrt{\pi}$.

   (c) Sum the series in part (a) to find an expression for $\phi(t)$ where

$$\phi(t) = \int_{\mathbf{R}} \cos(tx)\exp(-x^2)\,dx.$$

April 16, 2007