MA676 MWF 2-2:50pm CB 347 Spring 2007 Instructor: Russell Brown Office: POT741 Phone: 859 257 3951 russell.brown@uky.edu

Announcements

- 1. Our recitations will be held MR 11–11:50 in Mathskeller. Please try to attend at least once per week and be prepared to present the solution to one or more of the exercises.
- 2. The first set of problems is now due on Monday, 22 January 2007. A new set will be ready Monday.

EXERCISES

- 1. (Wheeden and Zygmund, p. 13) Compare $\limsup_{k\to\infty} a_k$ and $\limsup_{k\to\infty} (-\infty, a_k)$.
- 2. Suppose F_k , k = 1, 2, ... be a sequence of closed subsets of \mathbb{R}^n and we have $F_1 \supset F_2 \supset F_3 \ldots$ Is the intersection $\bigcap_{k=1}^{\infty} F_k$ nonempty?
- 3. Let

$$f(x) = \begin{cases} x \cos(\pi/x), & x \neq 0\\ 0, & x = 0 \end{cases}$$

Find V(f; 0, 1).

- 4. If [a, b] is a an interval and $f : [a, b] \to \mathbf{R}$, $g : [a, b] \to \mathbf{R}$ are functions, then show that $V(f + g; a, b) \le V(f; a, b) + V(g; a, b).$
- 5. In Sevilla, the barber shaves everyone who does not shave themself. Who shaves the barber of Sevilla?

PROBLEMS

These problems are due on Wednesday, 31 January 2007.

1. (Wheeden and Zygmund, p. 31) Let f_k be a sequence of functions which are of bounded variation on [a, b]. If $V(f_k; a, b) \leq M < \infty$ for all k and f_k converges to f pointwise, show that f is of bounded variation and $V(f; a, b) \leq M$.

Give an example of a sequence of functions which converge uniformly but so that the limit is not of bounded variation.

2. (Wheeden and Zygmund, p. 31) If $-\infty < \lambda_1 < \lambda_2 < \ldots < \lambda_m < \infty$ is a finite sequence and $\alpha_k \in \mathbf{R}$ for $k = 1, 2, \ldots, m$, express the sum

$$\sum_{k=1}^{m} \alpha_k \exp(-\lambda_k)$$

as a Riemann-Stieltjes integral.

January 23, 2007