

MA676
MWF 2-2:50pm
CB 347
Spring 2007

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ANNOUNCEMENTS

1. Our recitations will be held MR 11–11:50 in Mathskeller. Please try to attend at least once per week and be prepared to present the solution to one or more of the exercises.

EXERCISE SET 3.

1. Let α be continuous and of bounded variation on the interval $[0, 1]$. Find a simple expression which equals

$$\int_0^1 \alpha d\alpha.$$

2. Show that the rational numbers \mathbf{Q} have measure 0.
3. Show that $\mathbf{R} \times \{0\} \subset \mathbf{R}^2$ is a set of measure 0.
4. Show $|\mathbf{R}|_e = \infty$.
5. Find a collection $\{E_\alpha\}_{\alpha \in I}$ of sets so that each E_α is of measure zero but so that $|\cup E_\alpha|_e = \infty$.
6. Let $x_1 : [0, 1] \rightarrow \mathbf{R}$ and $x_2 : [0, 1] \rightarrow \mathbf{R}$ be continuously differentiable functions. Show that the image of (x_1, x_2) , $\{(x_1(t), x_2(t)) : 0 \leq t \leq 1\}$, is a set of measure zero. Hint: Show that each x_j is Lipschitz.

PROBLEM SET 3.

These problems are due on Monday, 5 February.

1. If f is a continuous function on the unit interval $[0, 1]$, prove that the graph of f , $\{(x_1, x_2) : x_2 = f(x_1), 0 \leq x_1 \leq 1\}$, has measure zero as a subset of \mathbf{R}^2 . Hint: Is f uniformly continuous?
2. Suppose that we imitate the construction of the Cantor set, but at each step we remove the open middle fifth from each subinterval. Find the outer measure of the resulting set.

January 29, 2007