ANNOUNCEMENTS

We seem to have more students at Monday recitation than on Thursday. If one or two volunteers would like to attend the Thursday session, I would appreciate it.

EXERCISE SET 4.

1. Let $f(x) = \cos(\pi/x)$ for $0 < x \le 1$ show that the graph of f is a set of measure zero. Hint: Can you give a soft argument that uses a previous problem?

2. Let $A \subset \mathbf{R}^n$ be a set. We define the outer Jordan content of A as

$$J(A) = \inf\{\sum_{j=1}^{N} v(I_j)\}$$

where the infinum is taken over all *finite covers* of A by intervals.

   (a) Show that $J(A) = J(\bar{A})$ where $\bar{A}$ is the closure of A.
   (b) Show that $|A|_e \le J(A)$.
   (c) Can you find a set A where we have $|A|_e < J(A)$?

3. (Stein, p. 41) Let E be the set of real numbers in $[0, 1]$ which do not have a decimal expansion containing a 7. Show E is measurable and find $|E|$.

PROBLEMS 4.

Due, Monday 12 February 2007.

1. Let E be a set in a metric space X with metric d. Define
   $d(x, E) = \inf\{d(x, y) : y \in E\}$ for $x \in X$.

   (a) Show that $d(x, E)$ is continuous. Hint: In fact, this function is Lipschitz with constant 1.
   (b) Show that for two sets A and B, $d(A, B) = \inf\{d(x, B) : x \in A\}$.
   (c) Let E be a set in a metric space. Show that if $d(x, E) = 0$, then x lies in the closure of E.
   (d) Suppose K is compact and F is closed and $K \cap F = \emptyset$. Show that $d(K, F) > 0$.

2. (Stein, p. 41)Suppose that $A \subset B \subset C$ and that A and C are measurable. If $|A| = |C| < \infty$, show that B is measurable.

February 4, 2007