MA676
MWF 2-2:50pm
CB 347
Spring 2007

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Announcement

We will try to have an exam the week of 26 Feb–2 Mar, as determined by the vote of the class. The exam will cover material through lecture 14.

Exercise set 6.

1. Show that for a set $E \subset \mathbf{R}^n$ and $x \in \mathbf{R}^n$, we have

$$|x + E|_e = |E|_e.$$

2. For a set $E \subset \mathbf{R}^n$ and s a real number, put $sE = \{sx : x \in E\}$. Give a direct proof that $|sE|_e = |s|^n|E|_e$.

3. Let $f : X \to Y$ be a function and let $\mathcal{N}$ be a σ-algebra on Y. Show that $\mathcal{M} = \{f^{-1}(B) : B \in \mathcal{N}\}$ is a σ-algebra on X.

4. If $f : \mathbf{R} \to \mathbf{R}$ is continuous and E is a Borel measurable set, is $f(E)$ Borel measurable?

5. Find an example of a set E which is in $\mathcal{L}$, the Lebesgue σ-algebra, and a continuous function f so that $f(E)$ is not Lebesgue measurable.

   Hint: Let f be the Cantor-Lebesgue function let $f(E) = N \cap f(C)$ where N is a non-measurable subset of the unit interval.

6. Let N be the non-measurable set constructed in lecture. Show that if $E \subset N$ is measurable, then $|E| = 0$.

Problem set 5.

1. Study for the exam.

February 20, 2007