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Exercise set 8.

1. Let $\{g_\alpha\}_{\alpha\in I}$ be a collection of continuous functions. Show that

$$h(x) = \sup\{g_\alpha(x) : \alpha \in I\}$$

is t semi-continuous. Determine if t equals upper or lower.

2. (Wheeden and Zygmund) Let $f : \mathbf{R}^n \to \mathbf{R}$ be a function. Fix a radius $r > 0$ and set

$$g(x) = \sup\{f(y) : y \in B(x, r)\}.$$

Show that g is lower semi-continuous.

3. (Wheeden and Zygmund, p. 62) Let $f : \mathbf{R} \to \mathbf{R}$ be continuous a.e. Show that f is measurable.

   Hint: Find a sequence of simple measurable functions which converge to f a.e. To do this, fix n, let $x_k = k/n$, $k \in \mathbf{Z}$ and set $f_n(x) = \sum_{k=-\infty}^{\infty} f(x_k^*)\chi_{[x_{k-1},x_k)}(x)$ for an appropriate choice of $x_k^* \in [x_{k-1}, x_k)$.

Problem set 8.

This problem should be handed in on Friday, 23 March 2007.

1. (Wheeden and Zygmund) Let $\{f_k\}_{k=1}^{\infty}$ be a sequence of Lebesgue measurable functions defined on a measurable set E with $|E| < \infty$. Suppose that for each x there is a number M_x so that $|f_k(x)| \le M_x$ for all k and all x in E.

   Show that for each $\epsilon > 0$, there is a closed set F with $F \subset E$, $|E \setminus F| < \epsilon$ and finite number M so that $|f_k(x)| < M$ for all $x \in F$ and all $k = 1, 2, \ldots$.

March 19, 2007