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Exercise set 9.

1. Find $a > 1$ so that
$$\int_1^a \frac{dt}{t} \geq 1.$$

2. Show that
$$\int_1^2 \frac{dt}{t} \leq 0.9$$
Hint: Remember that we know very little about the Lebesgue integral. If one does not know very much, is the exercise easier or harder?

3. Let $\{r_k\}_{k=1}^{\infty}$ be an enumeration of the rationals. Let $I_k = [r_k, r_k + 4^{-k}]$.
Is there an x for which the function $f(x) = \sum_{k=1}^{\infty} 2^k \chi_{I_k}(x)$ is finite?
Is there an open interval of $\mathbf{R}$ on which f is bounded?

4. (Hard?) (Stein, p. 46) Given an irrational number x, show that there are infinitely many rational numbers p/q with p and q relatively prime so that
$$|x - \frac{p}{q}| \leq \frac{1}{q^2}.$$
Hint: Use the pigeonhole principle.
Prove that the set of x in $\mathbf{R}$ for which there are infinitely many rational numbers p/q with p and q relatively prime and so that
$$|x - \frac{p}{q}| \leq \frac{1}{q^3}$$
is a set of measure zero. Hint: Use the Borel-Cantelli Lemma (see set 5).

5. Show that
$$\limsup_{n\to\infty} \cos(n) = 1.$$
You may assume that π is irrational.

Problem set 9.

These problems should be handed in on Friday, 30 March 2007.

1. If $\lim_{k\to\infty} f(t_k) = f(t)$ for every sequence $\{t_k\}$ with $t_k > t$ and $\lim_{k\to\infty} t_k = t$, prove that
$$\lim_{s\to t^+} f(s) = f(t).$$

2. Let f be a non-negative measurable function. Define the distribution function of f, ω_f by
$$\omega_f(t) = |\{f > t\}|.$$
Prove that
$$\lim_{s\to t^+} \omega_f(s) = \omega_f(t).$$
Do we have that $\omega_f(t)$ is continuous from the left? Hint: Consider our limit theorems for measure.

March 23, 2007