MA677
MWF 10-10:50pm
CB 345
Fall 2007

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The following problems will be due on 26 November 2007.

1. #16, page 191 of Wheeden and Zygmund.

2. Let $\mathcal{B}$ denote the Banach space $\mathbf{R}^2$ with the norm $\|x\| = |x_1| + 2|x_2|$. If $v \in \mathbf{R}^2$, then it is clear that $\lambda_v(x) = v_1 x_1 + v_2 x_2$ lies in the dual of $\mathcal{B}$. Find the norm of λ_v in the dual space $\mathcal{B}^*$.

3. Let B be a Banach space over $\mathbf{C}$ with norm $\|\cdot\|$. The Hahn-Banach theorem tells us that if $\ell : V \to \mathbf{C}$ is a linear map on a subspace V of B and $|\ell(f)| \le C\|f\|$ for all $f \in V$ and some $C < \infty$, then we may find an extension $\tilde{\ell} : B \to \mathbf{C}$ so that $|\tilde{\ell}(f)| \le C\|f\|$ for all $f \in B$.

   (a) Consider $L^1(\mathbf{R})$ and $L^\infty(\mathbf{R})$ with respect to Lebesgue measure. Use the Hahn-Banach theorem to show that the dual of $L^\infty(\mathbf{R})$ is not $L^1(\mathbf{R})$. To do this define $\delta : C_c(\mathbf{R}) \to \mathbf{C}$ by $\delta(\phi) = \phi(0)$ and use the Hahn-Banach theorem to construct an extension $\tilde{\delta} : L^\infty(\mathbf{R}) \to \mathbf{C}$.

   (b) Show that we cannot find a function in L^1 so that
   $$\delta(\phi) = \int_{\mathbf{R}} f(x)\phi(x)\,dx.$$
   Hint: Suppose such an f exists and show that f must vanish a.e.

   (c) (Extra credit) Show that there are uncountably many different extensions of δ to L^∞.

November 14, 2007