MA677 MWF 10-10:50pm CB 345 Fall 2007 Instructor: Russell Brown Office: POT741 Phone: 859 257 3951 russell.brown@uky.edu

The following problems will be due on 19 September.

1. Provide a proof of Hölder's inequality from Jensen's inequality.

Thus for $g \ge 0$ and $f \ge 0$ and $1 < q < \infty$ suppose q' = q/(q-1). In the special case where g(x) > 0 for all x and $||g||_q = 1$, apply Jensen's inequality where the weight (the function p) is g^q and f is replaced by f/g^{q-1} . Deduce the general case from this special case.

2. (Wheeden and Zygmund, p. 124) Suppose that $\phi : \mathbf{R} \to \mathbf{R}$ is continuous and for all real numbers x and y,

$$\phi(\frac{x+y}{2}) \le \frac{1}{2}\phi(x) + \frac{1}{2}\phi(y).$$

Prove that ϕ is convex.

- 3. (Not graded) Does the result of the previous problem hold if ϕ is not assumed to be continuous?
- 4. (Wheeden and Zygmund, p. 143) Prove the converse of Hölder's inequality when p = 1 and when $p = \infty$. You may assume the functions are real-valued.
- 5. Show that Minkowski's inequality may fail if p < 1.

September 10, 2007