MA677 MWF 10-10:50pm CB 345 Fall 2007 Instructor: Russell Brown Office: POT741 Phone: 859 257 3951 russell.brown@uky.edu

The following problems will be due on 5 October.

- 1. Suppose X is a separable Hilbert space and  $A \subset X$  is a set that satisfies ||x y|| = 1 for  $x, y \in A$  and  $x \neq y$ . Show that A is at most countable.
- 2. Let p satisfy  $1 and suppose that f is in <math>L^p(E)$ . Define

$$g(x) = \begin{cases} f(x), & |f(x)| \le M\\ 0, & |f(x)| > M \end{cases}$$

For which q do we have  $g \in L^q(E)$ ?

3. (Riemann Lebesgue Lemma) Suppose that f is in  $L^1([0, 2\pi])$ . Prove that

$$\lim_{k \to \infty} \int_0^{2\pi} f(x) e^{-ikx} \, dx = 0.$$

Hint: We know this holds if f is in  $L^2$ . Can you approximate an  $L^2$  function by an  $L^1$  function?

4. Let  $\{f_k\}$  be a sequence in  $L^p(E)$ . We say that the sequence  $\{f_k\}$  converges weakly to f if

$$\lim_{k \to \infty} \int f_k g \, dx = \int f g$$

for all g in  $L^{p'}$ . Suppose that  $\{f_k\}$  converges to f in  $L^p$ . Show that  $\{f_k\}$  converges weakly to f.

5. Suppose that  $\{f_k\}$  is a sequence in  $L^2$  and that  $f_k$  converges weakly to f. Show that if  $||f_k||$  converges to  $||f||_2$ , then we have that  $\{f_k\}$  converges in  $L^2$ .

Can you find an example where  $\{f_k\}$  converges weakly to 0, but  $||f_k||_2 = 1$  for all k?

October 2, 2007