MA677
MWF 10-10:50pm
CB 345
Fall 2007

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The following problems will be due on 12 November 2007.

1. Let Σ be the σ-algebra on the real line which is generated by the intervals $[k, k+1)$, for $k \in \mathbf{Z}$. Is the function given by $f(x) = x$ measurable for this σ-algebra?

2. (Wheeden and Zygmund, p. 191, #13) Let ϕ be a real-valued additive set function on $(\mathcal{S}, \Sigma)$. Let P_1 and P_2 be two sets as in the Hahn-decomposition for ϕ. Show that the symmetric difference, $P_1 \Delta P_2$, is a null set. We say that E is a null set for an additive set function ϕ, if we have $\phi(A) = 0$ for all measurable subsets A of E.

3. (Extra credit) Let f be in $L^2(\mathbf{R})$ and for (x, y) in $\mathbf{R}^2$ with $y > 0$, define the Poisson integral of f by
$$u(x, y) = \frac{1}{\pi} \int_{\mathbf{R}} f(t) \frac{y}{y^2 + (x-t)^2} \, dt.$$
Provide a careful proof that $D_y u(x, y)$ exists.

November 1, 2007