Recitation 12 MA113:004–006
6 October 1998 Fall 1998

Reminders: 1. A notebook is on reserve in the math library for this course. This notebook contains tests from last year and solutions to tests, homework, quizzes and projects. The math library is located in the basement of the Patterson Office Tower.
2. The next homework is due on Wednesday, 7 October. The assignment is §2.3 #12, 20 §2.4 #6, 14.
3. We are one lecture behind the schedule which was handed out for section 2. Thus, you will probably not get very far in this worksheet on Tuesday. Section 2.6 is fairly short, thus I expect that we will catch up by the end of this week.

Below is a selection of problems related to sections 2.5. These problems will not be collected or graded. However, you should understand how to work each of these problems. You should begin working on these problems in groups in recitation. You will probably want to finish these problems outside of class. If you have questions, please ask your TA or instructor. If you find a problem difficult, consider working similar problems from the text for additional practice.

1. (Algebra review) Obtain a common denominator and simplify the numerator.
$$\frac{1}{a+b} - \frac{1}{a}.$$
2. (Algebra review) Obtain a common denominator and simplify the numerator.
$$\frac{1}{(a+b)^2} - \frac{1}{a^2}.$$
3. Work the following exercises from Section 2.5. #3, 5, 7, 15, 17, 19, 21, 25, 29, 33, 35, 41.
4. What is $\infty - \infty$? We have not defined arithmetic with the symbol ∞. One way to carry out arithmetic with infinity is to find a function $f(x)$ so that $\lim_{x\to a} f(x) = \infty$. Then carry out arithmetic with the function $f(x)$ and take the limit. Sometimes this works well. For example, it can be seen that $\infty + \infty = \infty$ or that for any real number c, $\infty + c = \infty$. Sometimes, however, this does not work. The problem is that if you make different choices for the function f, then you might get different answers for $\infty - \infty$. In this problem, you are asked to produce examples to show that we cannot make a sensible definition of $\infty - \infty$.

   This also explains why I insist that if a limit is ∞, then it is undefined (but is ∞). This way, you will not be tempted to apply the limit laws to limits which take the value ∞.

   For the problems below, let $f(x) = 1/x^2$.

(a) Consider $f(x)$, $g(x) = f(x)$ and $f(x) - g(x)$ and compute

$$\lim_{x\to 0} f(x), \qquad \lim_{x\to 0} g(x), \qquad \lim_{x\to 0} f(x) - g(x).$$

Based on your answers, what should $\infty - \infty$ be.

(b) Consider $f(x)$, $g(x) = 73 + f(x)$ and $f(x) - g(x)$ and compute

$$\lim_{x\to 0} f(x), \qquad \lim_{x\to 0} g(x), \qquad \lim_{x\to 0} f(x) - g(x).$$

Based on your answers, what should $\infty - \infty$ be.

(c) Can you give an example where it appears that $\infty - \infty$ should equal 24?