MA114:012–014 Russell Brown

Final 8 May 1998

Answer all of the following questions. Use the backs of the question papers for scratch paper. Additional sheets are available if necessary. No books or notes may be used. When answering these questions, please be sure to 1) check answers when possible, 2) clearly indicate your answer and the reasoning used to arrive at that answer **(unsupported answers may receive NO credit)**, and 3) label all variables and equations.
In general, you must use the methods introduced in this course to solve problems. In particular, no credit will be given for evaluating an integral using your calculator.

Name ____________________

Section __________

Question	Score	Total
1		15
2		20
3		15
4		15
5		15
6		20
Total		100

1. Use calculus to evaluate the following integrals.

   (a) $\int_0^2 xe^{-x^2}\,dx$

   (b) $\int_0^2 xe^{-x}\,dx$

   (c) $\int_0^1 \frac{1}{2+3x}\,dx$

2. Consider a right-circular cone whose base has radius $r = 3$ cm and height $h = 6$ cm. Carry out the following steps to find the volume of the cone.

   (a) If we slice the cone perpendicular to its axis and at a distance of x cm from the vertex, give the area of the cross-section.

   (b) We make equally spaced slices perpendicular to the axis in order to produce N piece. Approximate each of these pieces by a slab whose cross section has area equal to the bottom of the piece. Write down the sum of the volumes of the N slabs as an approximation to the total volume of the cone.

   (c) Take the limit as $N \to \infty$ of the sum in part b) to obtain an integral which gives the volume of the cone.

   (d) Evaluate the integral in part c).

3. (a) Find a function $y(x)$ which satisfies the following conditions:

$$y' = 4 - y, \qquad y(0) = 3.$$

(b) Find $\lim_{x\to\infty} y(x)$.

(c) Find all equilibrium solutions of the equation $y' = 4 - y$. Recall that an equilibrium solution is a solution which is constant, $y(x) = c$.

4. Let $f(x) = e^{-2x}$.

(a) Give a formula for the nth derivative of f, $f^{(n)}(x)$.

(b) Write out the MacLaurin series for $f(x)$.

5. (a) Express

$$\frac{1}{1+x^2}$$

as a power series.

(b) State the theorem on differentiation and integration of power series.

(c) Find the MacLaurin series for $\tan^{-1} x$.

(d) For which x does the series you found in part c) converge?

6. This problem asks you to find the power series for $1/(1+x)^2$ in two ways. The best papers will find that both methods give the same answer.

   (a) Write
   $$\frac{1}{1+x}$$
   as a series in powers of x.

   (b) Differentiate the series in part a) termwise and give the series for
   $$\frac{1}{(1+x)^2}.$$
   For which x does this series converge?

   (c) Use the binomial series to find the MacLaurin series for
   $$\frac{1}{(1+x)^2}$$
   .