Calculus II — Russell Brown

Review 1 — January 28, 1999

As announced, the test will be on Tuesday, 9 February 1999 from 7:30 to 9:30. The room for the tests will be announced in class, once it is determined.

You may use a graphing calculator during the exam. You may not use notes, the text or any sort of integral table on the exam. You will be expected to know all of the integrals in the table on page 375. (Didn't you learn them last semester as differentiation rules?) On the exam, you should try to explain your work as carefully and completely as you can. Students giving unsupported answers may not receive full credit even if they give the correct answers. Students who present their work neatly and carefully may receive more partial credit than students whose papers are hard to read. If I can't read it, I can't give you credit for it!

The test will cover sections 1–6, 8 and 9 of Chapter 5. Below are several questions to help review for the exam. As you review for the exam, you should make sure that you can work all assigned problems from the text, quizzes and review information presented in class.

1. Find numbers a and b which satisfy
$$a < \int_1^3 \frac{dt}{t} < b \text{ and } b - a < 1.$$

2. Suppose that t represents the time in hours after twelve noon on 11 December 1996 and one sees $g(t)$ birds per hour while birdwatching. What does
$$\int_2^4 g(t)\,dt$$
represent?

3. Find a function that satisfies $F'(x) = x^2 + 2$ and $F(2) = 2$.

4. Evaluate
   (a) $\displaystyle\int_1^2 \frac{dt}{t}$
   (b) $\displaystyle\int \frac{x}{4+x^2}\,dx$
   (c) $\displaystyle\int \frac{x^2}{4+x^2}\,dx$
   (d) $\displaystyle\int \frac{x+2}{x^2+4x+5}\,dx$
   (e) $\displaystyle\int \frac{1}{x^2+4}\,dx$
   (f) $\displaystyle\int e^x \cos x\,dx$

(g) $\int_0^1 x^2 \cos x \, dx$

(h) $\int_0^1 x \cos x^2 \, dx$

(i) $\int_{-1}^1 x e^{x^2} \, dx$

5. State and prove part 1 of the fundamental theorem of calculus. State and prove the evaluation theorem. Be sure that you understand all results used in the proofs of these theorems.

6. Differentiate the function $F(x)$ which is defined by

$$F(x) = \int_{x^2}^{x^3} \cos t \, dt.$$

7. If $f(x)$ is linear, $f(2) = 3$ and $f(4) = 10$, find

$$\int_2^4 f(t) \, dt.$$

Can you answer this question without finding a formula for f?

8. Evaluate the integral using a geometric argument

$$\int_1^2 \sqrt{4 - x^2} \, dx.$$

9. If $\int_0^2 f(x) \, dx = 2$, what is $\int_0^1 f(2x) \, dx$?

10. Use the Trapezoid rule, Simpson's rule and the Midpoint rule with $n = 4$ to approximate $\ln 3$. What is the upper bound for the error in Simpson's rule? What is the actual error?

11. How many terms in Simpson's rule are needed to compute

$$\int_0^1 \sin(2x) \, dx$$

to within an accuracy of 10^{-4}?

12. Find the following integrals, if they exist.

(a) $\int_0^2 \frac{1}{x-1} \, dx$

(b) $\int_1^\infty \frac{1}{1+x^2} \, dx$

(c) $\int_{-1}^1 |x|^{-1/2} \, dx$